

# **HICUM Workshop**

**June 6/7, 2002**

## **Improved $T_f$ and $f_T$ determination**

prepared by

Markus Malorny

Chair for Electron Devices and Integrated Circuits (CEDIC)  
University of Technology Dresden, Germany

## Overview

- Introduction
- Parameter extraction according to the conv. method (CM)
- Improved method (IM) - theory
- Parameter extraction according to the IM
- Deviation of the transit frequency in the high-current region
- Summary

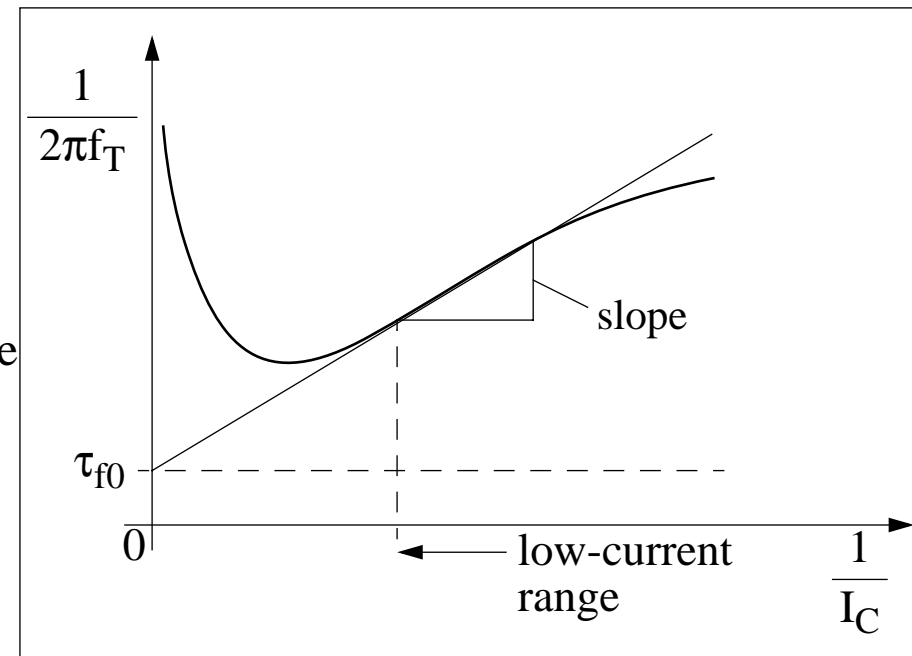
## Introduction - The conventional method (CM)

- relation between transit frequency and transit time:

$$\frac{1}{2\pi f_T} = \tau_f(V_{BC}, I_C) + (r_{Cx} + r_E)C_{jC}(1 + 1/\beta_0) + \frac{C_{jC}(1 + 1/\beta_0) + C_{jE}V_T m_{Cf}}{I_C}$$

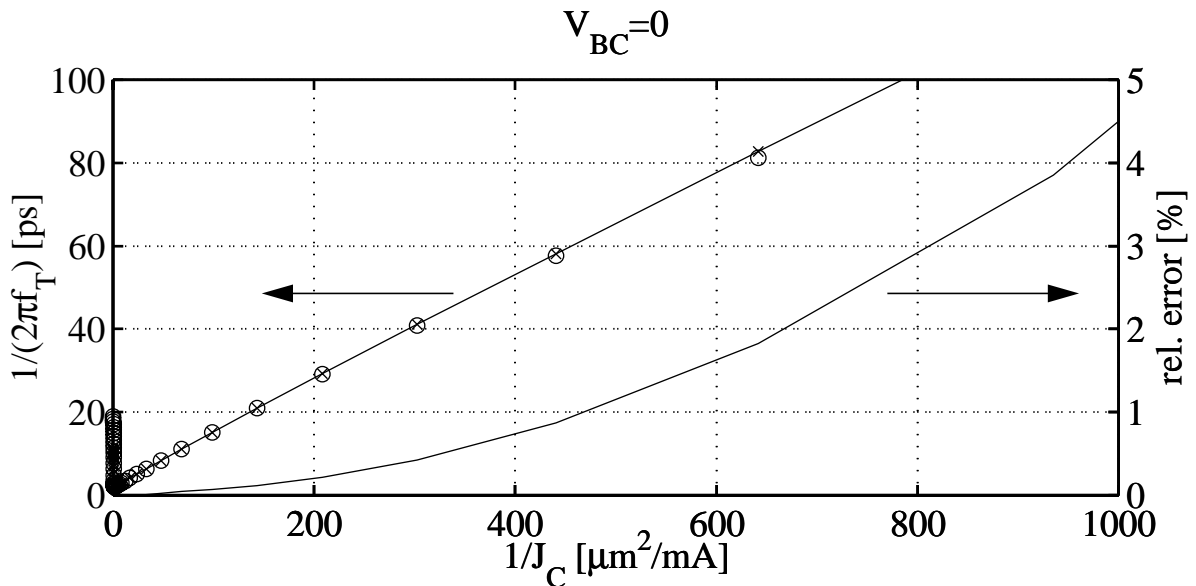
- CM for extracting transit time:

- find inflection point in plot  $\frac{1}{2\pi f_T}$  vs.  $\frac{1}{I_C}$
- calculate a tangent at the inflection point
- $\tau_{f0}$  = intersection of tangent with ordinate
- calculate  $C_{jE,max}$  from the tangent slope

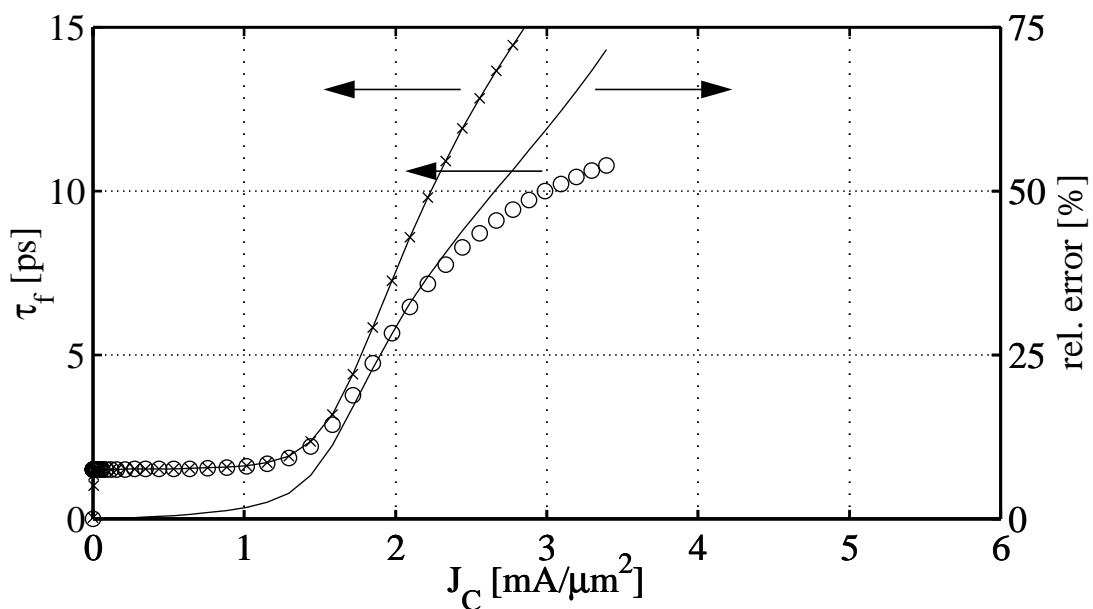


## Verification of the transit frequency relation

- low-current region: the equation approximates the reference



- high-current region: equation does not approximate the reference



$\Rightarrow$  equation has to be revisited for high-current region

## Why using simulation results instead of measurements to verify an extraction methodology?

### Advantages:

- model parameters are known: extracted result is easily comparable with parameter setup
- deviation of extracted result is only due to the used methodology and not due to measurement uncertainty

### Disadvantages:

- applicability to measured data still must to be verified

## Extraction result according CM at low-current densities

- the rel. error is in the range of 10%

Discussion:

- various causes for inadequacy of the CM (exact result depends on process)

⇒ extracted result with CM is random

- 1st example:

$$\text{proc 1: } C_{jE}(I_{C1}) = C_{jE, max} \text{ and } \Delta\tau_f(I_{C1}/I_{CK1}) > 0$$

$$\text{proc 2: } C_{jE}(I_{C1}) = C_{jE, max} \text{ and } \Delta\tau_f(I_{C1}/I_{CK2}) = 0$$

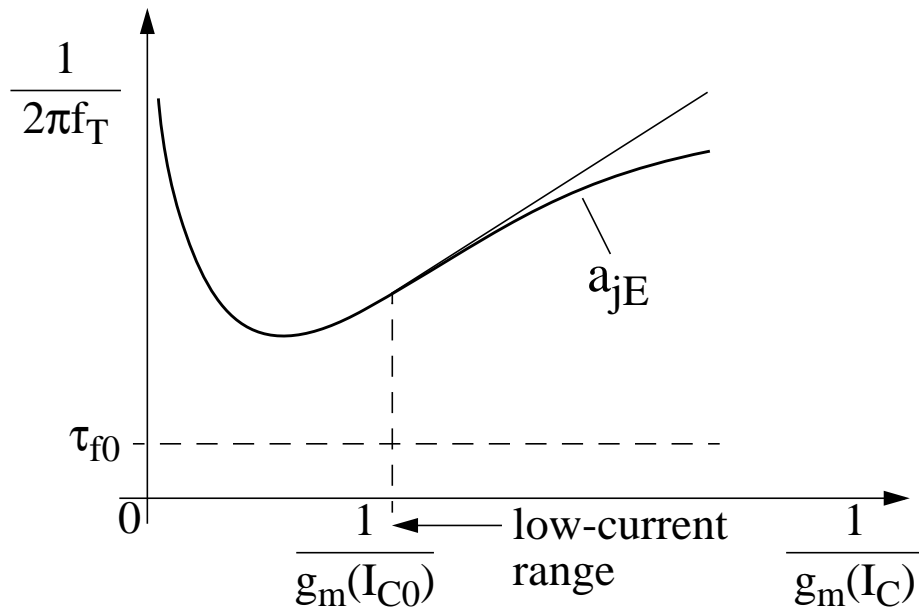
$I_{C1}$  collector current in inflection point,  $I_{CK1}$  and  $I_{CK2}$  critical current at  $V_{CE1} = V_{CE2}$ .

⇒ calculating  $C_{jE, max}$  in proc 1 from slope is impossible because of influence of  $\Delta\tau_f$  and  $I_{CK1} < I_{CK2}$

- 2nd example: same setup as in example 1

⇒ a deviation in the slope will cause a deviation in the extrapolated value of  $\tau_{f0}$

## Improved method for extracting $\tau_f$ at low-current densities



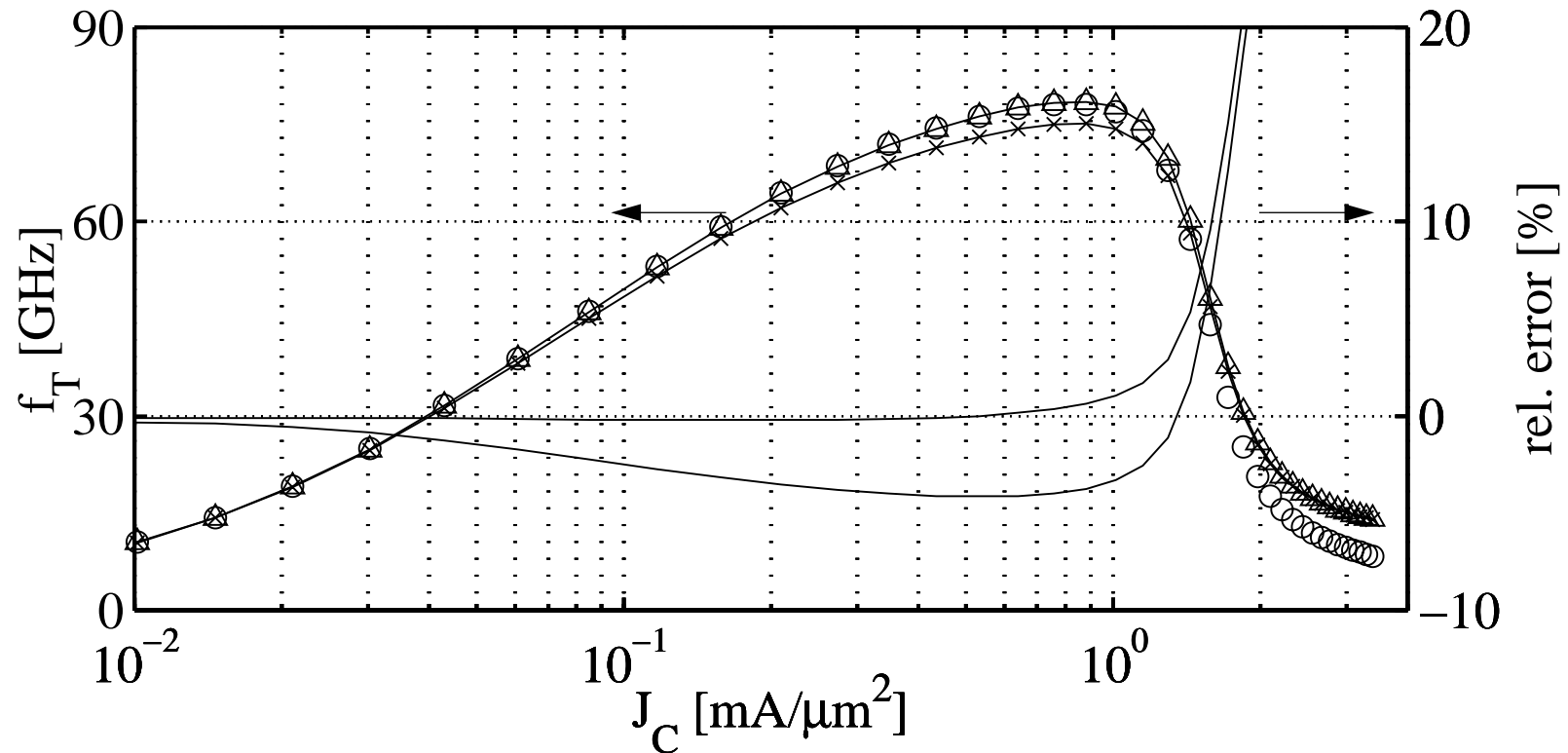
$$g_m = \frac{\Re(y_{21})}{1 - r_E \Re(y_{21})}$$

Improved method:

- determine  $I_{C0}$  from tangent in inflection point  
 $\Rightarrow$  lower bound
- fit equation at sufficiently low-current densities  
 $\Rightarrow$  parameters  $\tau_{f0}$ , and  $a_{jE}$

$$\left. \frac{1}{2\pi f_T} \right|_{\frac{1}{g_m} \geq \frac{1}{g_m(I_{C0})}} = \tau_{f0} + \frac{C_{jC} + a_{jE} C_{jE0} f(V_{BE})}{g_m(I_C)} \left. \right|_{\frac{1}{g_m} \geq \frac{1}{g_m(I_{C0})}}$$

## Comparison of the extracted results

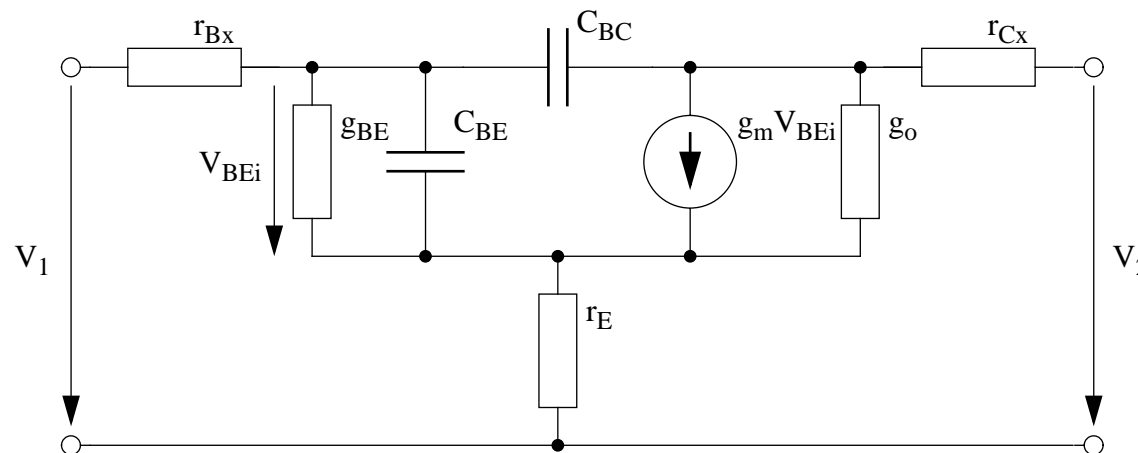


Discussion:

- more accurate result than in case of extraction according CM
- remaining deviation at high-current densities caused by inadequate standard equation for  $1/(2\pi f_T)$

## Transit frequency relation including external elements

- small signal equivalent circuit



$$C_{BC} = C_{jC} + C_{dC}$$

$$C_{BE} = C_{jE} + C_{dE}$$

$$C_{dC} = \tau_f g_o$$

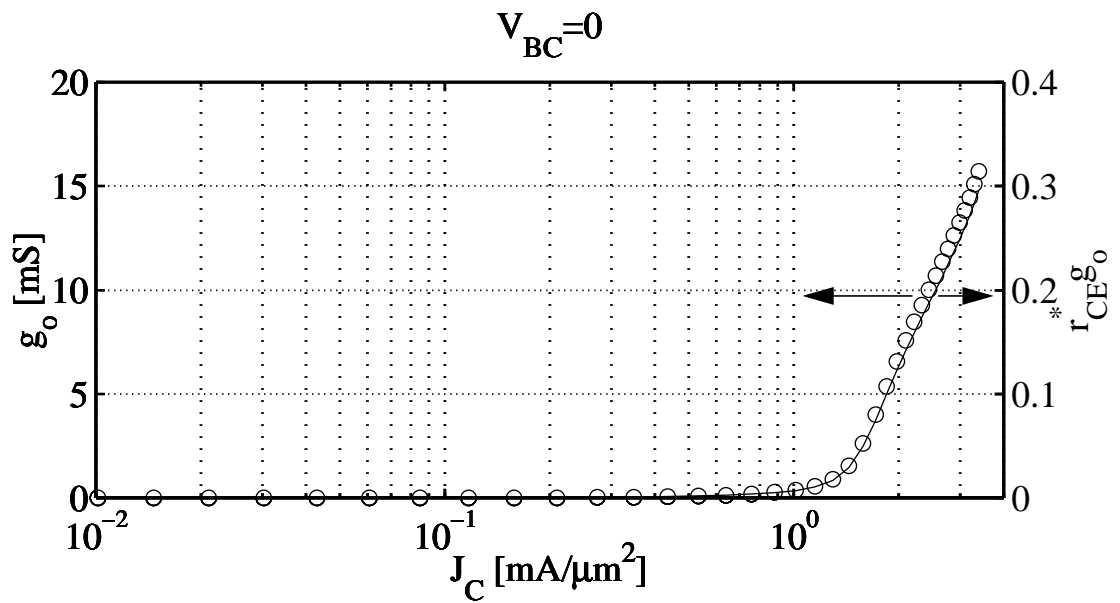
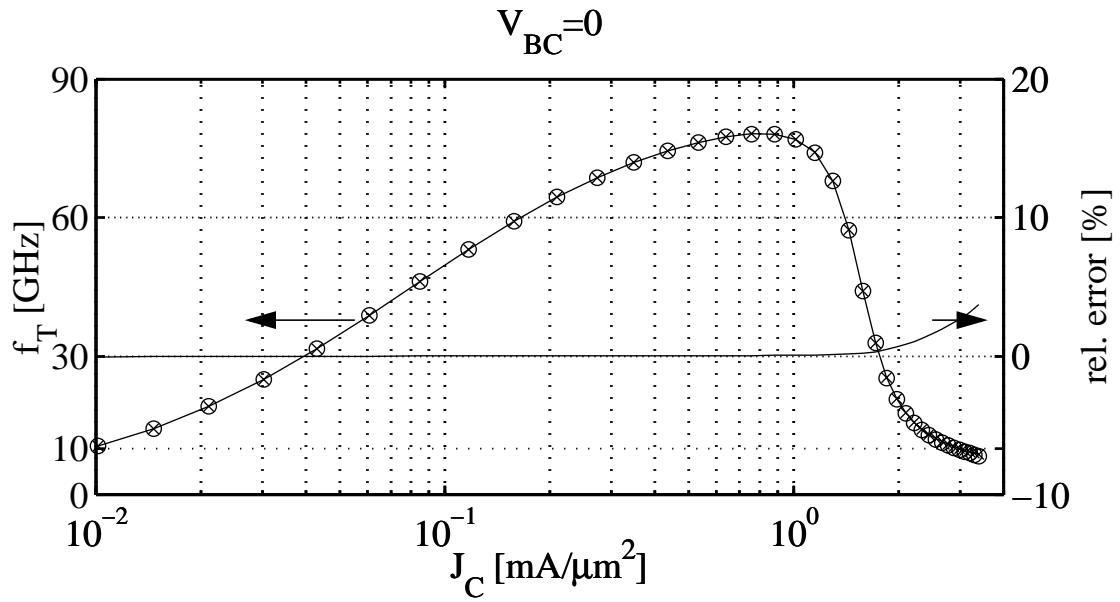
$$C_{dE} = \tau_f g_m$$

- simplified equation (from symbolic analysis)

$$\frac{1}{2\pi f_T} = \tau_f(1 + r_{CE}^* g_o) + r_{CE}^* C_{BC}(1 + 1/\beta_0) + \frac{(C_{jE} + C_{BC})(1 + r_{CE}^* g_o) + C_{BC}1/\beta_0}{g_m}$$

$$r_{CE}^* = r_{Cx} + r_E(1 + 1/\beta_0)$$

## Verification of the new equation



## Using the new equation

- inserting  $C_{BC} = C_{jC} + C_{dC}$ ,  $C_{dC} = \tau_f g_o$  in the new eq. and a rearrangement leads to

$$\frac{1}{2\pi f_T} = \tau_f \left( 1 + r_{CE}^* g_o \left( 2 + \frac{1}{\beta_0} + \frac{g_o}{g_m} \right) + \left( 1 + \frac{1}{\beta_0} \right) \frac{g_o}{g_m} \right) + r_{CE}^* C_{jC} \left( 1 + \frac{1}{\beta_0} \right) + \frac{(C_{jE} + C_{jC})(1 + r_{CE}^* g_o) + C_{jC}/\beta_0}{g_m}$$

- with exception of  $g_m$  and  $g_o$  all values are known from measurements or previous extraction steps (same as in CM).

- $g_m = \frac{\Re(y_{21})}{1 - r_E \Re(y_{21})}$  and  $g_o = \Re(y_{22})$  at low frequencies

$$\frac{1}{2\pi f_T} = \tau_f f(g_o, g_m) + h(g_o, g_m, a_{jE}) = \tau_{f0} f(g_o, g_m) + \Delta\tau_f f(g_o, g_m) + h(g_o, g_m, a_{jE}) \quad (*)$$

$$\left. \frac{1}{2\pi f_T} \right|_{\frac{1}{g_m} \geq \frac{1}{g_m(I_{C0})}} = \tau_{f0} f(g_o, g_m) + h(g_o, g_m, a_{jE}) \left| \frac{1}{g_m} \geq \frac{1}{g_m(I_{C0})} \right. \quad (**)$$

- use (\*\*) in low-current range and fit simultaneously the parameters  $a_{jE}$ ,  $\tau_{f0}(\tau_0, \Delta\tau_0, \tau_{BfvI})$
- calculate  $\Delta\tau_f$  from (\*) and fit the high-current component of the transit time

## Summary

- The conventional method for extracting the transit time is inadequate (rel. error in the order of 10% and 60% at low to high current densities)
- An improved method has been suggested and applied on simulated data (rel. error < 0.5% )
- In high-current region the influence of  $g_o$  and  $C_{dC}$  can be significant.
- A new simplified equation has been shown and successfully verified (rel. error < 4% )