Two-/three-dimensional GICCR Master equation for Si/SiGe bipolar transistors

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Introduction

Bipolar transistor applications

- high-frequency/high-speed operation
  - Bluetooth, 802.11, WLAN (e.g. 60GHz), UWB (impulse) radio, Free Space Optics
  - OC 192/768...
  - linear circuits, ...

Constraints

- some applications are close to the technology limit $\Rightarrow$ careful circuit optimization
- cost reduction (mask, re-spins..., yield prediction)

$\Rightarrow$ need for accurate compact models for SiGe HBTs

- presently: covered by advanced models such as HICUM, MEXTRAM, VBIC
- future: increasing importance of accurate models (e.g., geometry scaling ...)

$\Rightarrow$ **goal**: physics-based model concept for future technologies

Transfer current theory

- classic approach: differential equation in base region
  - 1D
  - different formulation for each bias and spatial region (no single-piece expression)
  - important effects (such as Early- and high-current effect) are difficult to include

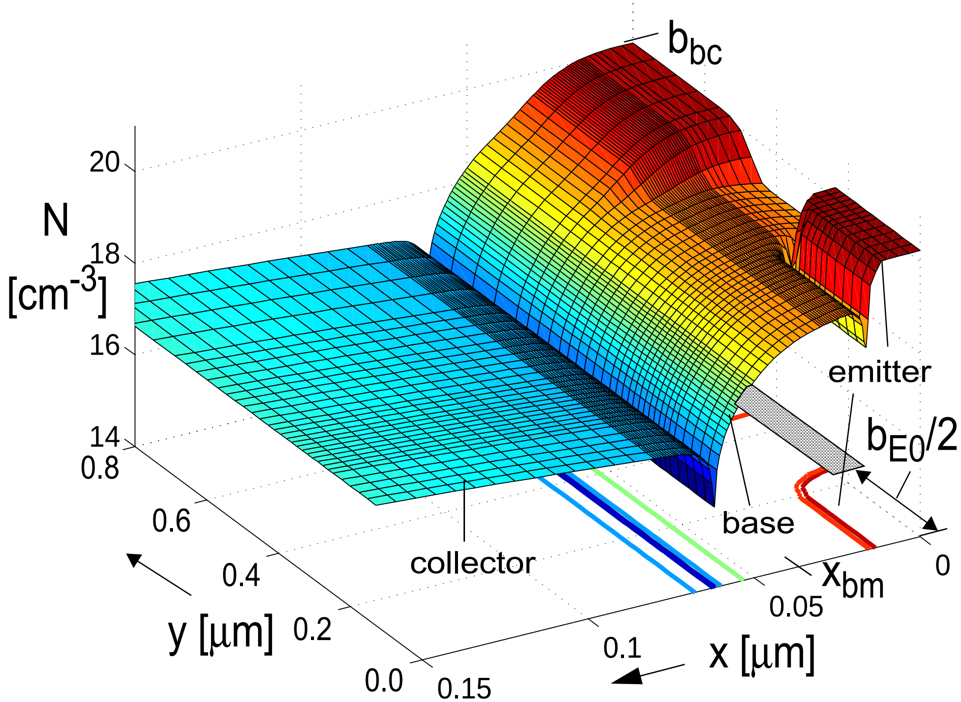
- Integral Charge-Control Relation (ICCR):
  - natural to **BJT** action, elegant single-piece solution over relevant bias region
  - requires smooth charge model
  - limited to transistors without bandgap variation, 1D structure

- Generalized ICCR (GICCR)
  - extension of ICCR
  - applicable to HBTs, but still 1D

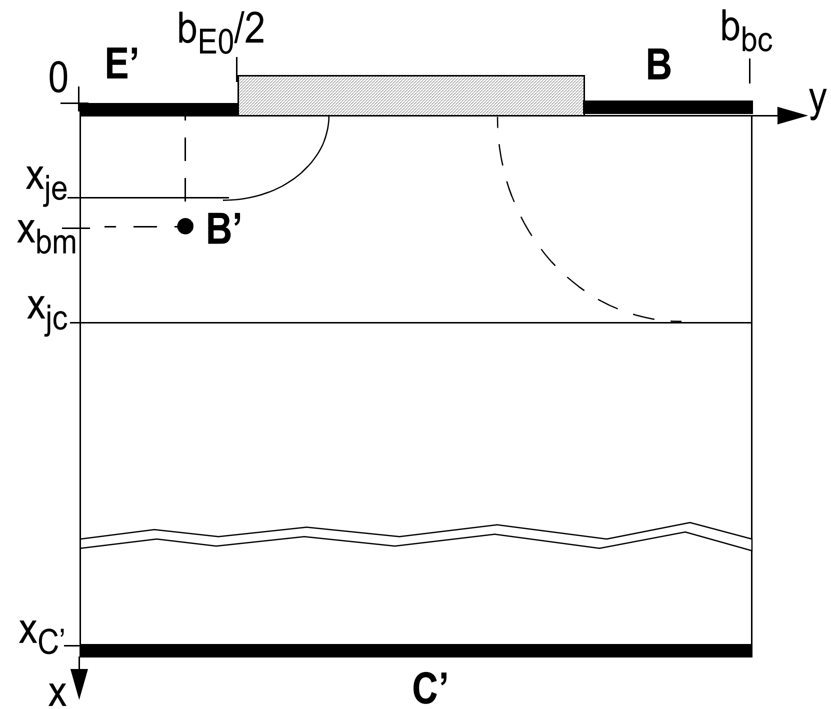
$\Rightarrow$ This work: multi-dimensional GICCR formulation

Investigated device structure and process technology

vertical doping profile (1D) under emitter



schematic cross-section of SiGe HBT



- profile corresp. to high-speed transistor version with peak $f_T \approx$ 100GHz @ V_{BC} = 0V
- example here: "conventional" profile (other profiles have been investigated, too)
- 2D device simulation: unit emitter length l_E = 1µm (no new information by 3D simul.)

Theory and derivation

- assumptions:
  - negligible volume recombination
  - zero time derivative (for practical applications: quasi-static operation)
  - tunneling and thermionic emission across the junctions are neglected

- starting point: x-component of electron current density, $J_{nx} = -q\mu_n n \frac{d\varphi_n}{dx}$

- yields (same as 1D derivation): $I_T\, h(x,y) p(x,y) = -c_0 \frac{d[\exp(-\varphi_n/V_T)]}{dx} \exp\left(\frac{V_{B'E'}}{V_T}\right)$

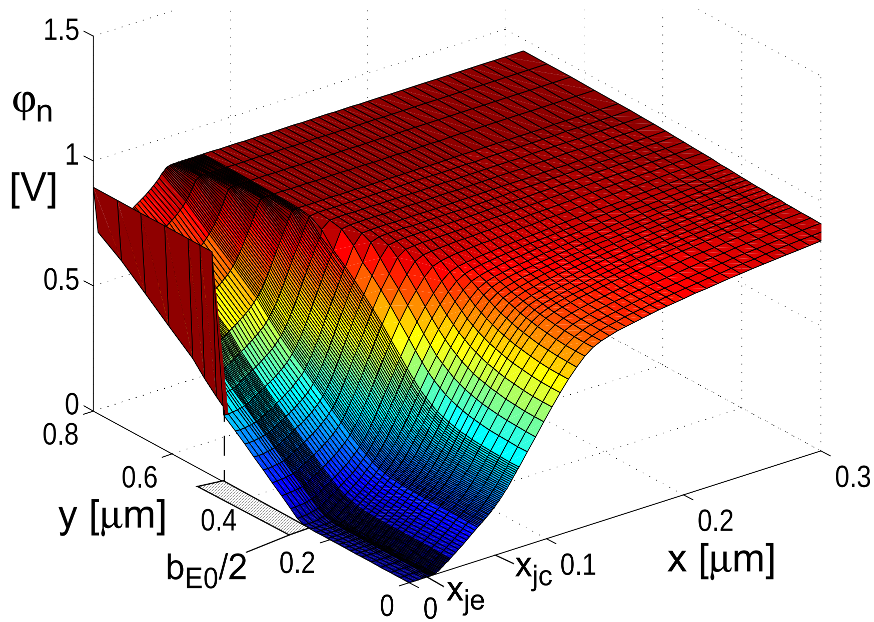
- with constant $c_0 = qV_T A_{E0} \mu_{n0} n_{i0}^2$ and weighting function

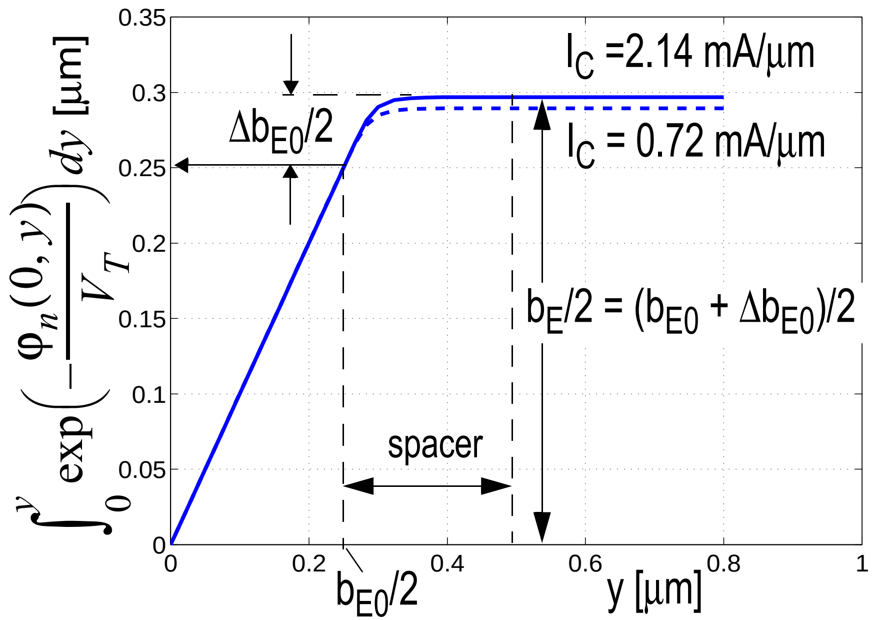
$$h(x,y) = \underbrace{\frac{-J_{nx}(x,y)}{I_T/A_{E0}}}_{\mathrm{h_J(x,y)}} \underbrace{\frac{\mu_{n0} n_{i0}^2}{\mu_n n_i^2}}_{\mathrm{h_g(x,y)}} \underbrace{\exp\left(\frac{V_{B'E'} - \varphi_p(x,y)}{V_T}\right)}_{\mathrm{h_e(x,y)}}$$

Theory (cont'd)

- 2D integration of r.h.s.:

$$\int_0^{b_{bc}} \int_{\exp\left(-\frac{\varphi_n(0,y)}{V_T}\right)}^{\exp\left(-\frac{\varphi_n(x_{C'},y)}{V_T}\right)} d\left[\exp\left(-\frac{\varphi_n}{V_T}\right)\right] dy = \int_0^{b_{bc}} \exp\left(-\frac{\varphi_n(x_{C'},y)}{V_T}\right) dy - \int_0^{b_{bc}} \exp\left(-\frac{\varphi_n(0,y)}{V_T}\right) dy$$



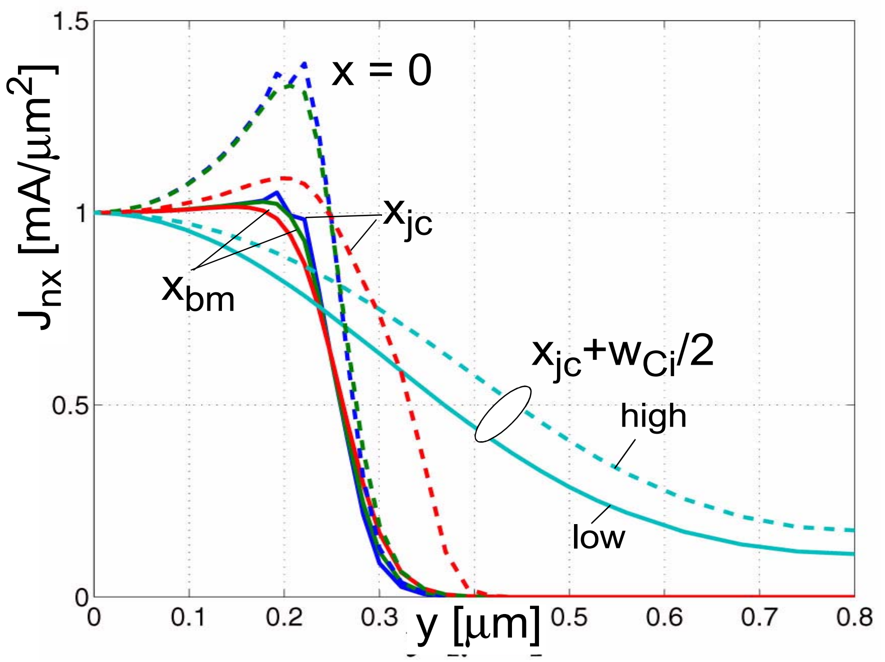


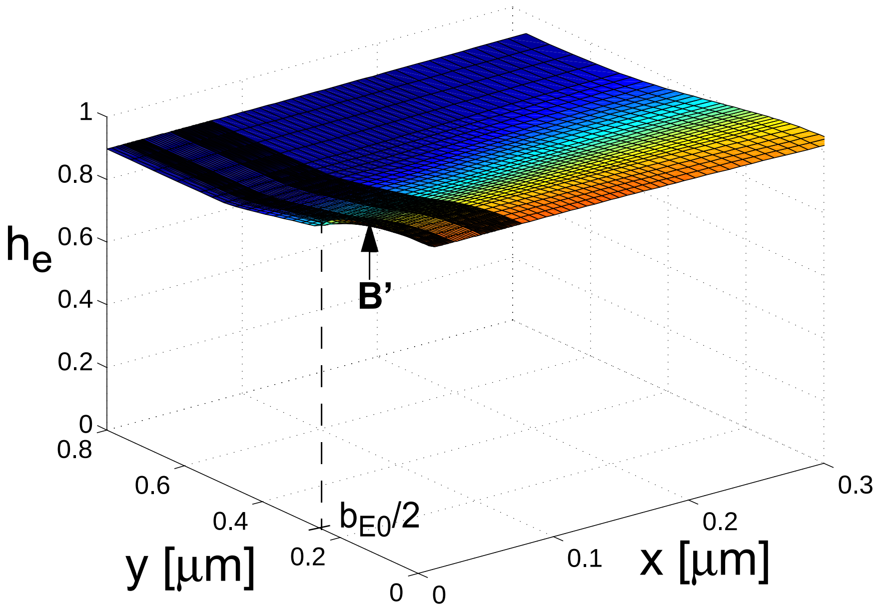
$$\int_0^{b_{bc}} [\ldots] dy = \exp\left(-\frac{V_{C'E'}}{V_T}\right) b_{bc} - \left(\frac{b_{E0}}{2} + \frac{\Delta b_{E0}}{2}\right) \quad \text{with} \quad \Delta b_{E0} = 2\int_{b_{E0}/2}^{b_{bc}} \exp\left(-\frac{\varphi_n(0,y)}{V_T}\right) dy$$

Theory (cont'd)

- 2D integration of l.h.s.: $I_T \int_0^{b_{bc}} \int_0^{x_{C'}} h_g(x,y)\; h_J(x,y)\; h_e(x,y)\; p(x,y)\; dxdy$
- $h_g = (\mu_{n0} n_{i0}^2)/(\mu_n n_i^2)$: impact in y-direction negligible $\Rightarrow$ treat as in 1D case

$$h_J = \frac{-J_{nx}(x,y)}{I_T / A_{E0}} \qquad h_e = \exp\left(\frac{V_{B'E'} - \varphi_p(x,y)}{V_T}\right)$$



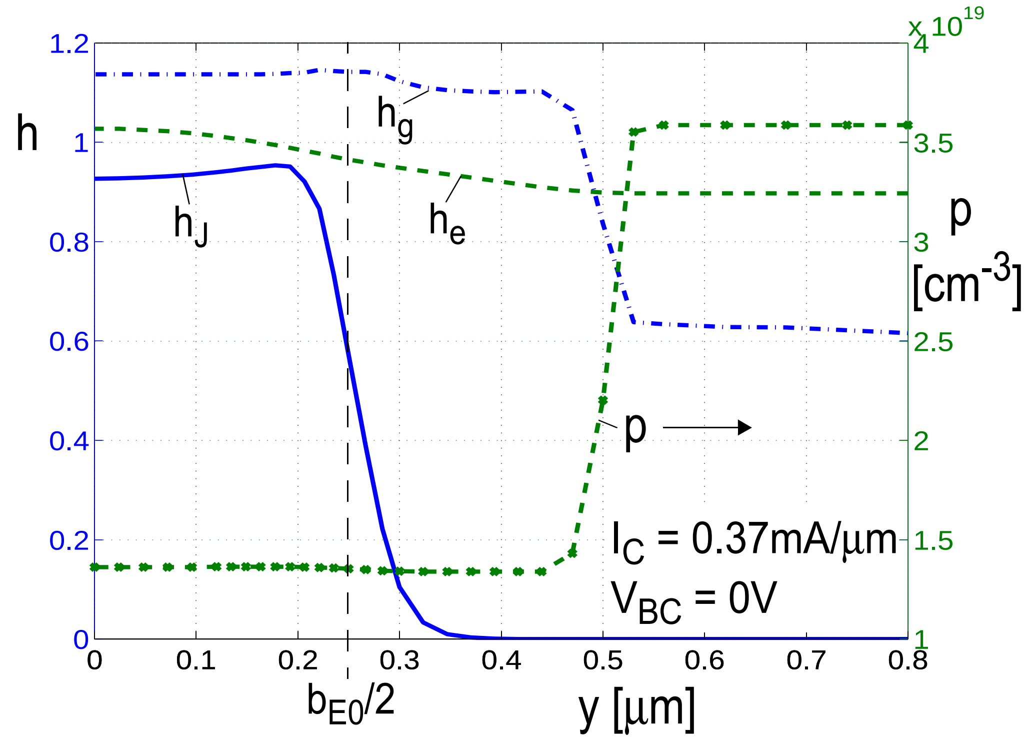


Theory (cont'd)

- relation of weighting functions to hole density distribution

lateral spatial dependence of h_J, h_e, h_g, and p along $x = x_{bm}$

- h_g
  - constant vs y $\Rightarrow$ $h_g \approx h_g(x,0)$
- h_J
  - constant vs y under most of window (can be affected by emitter current crowding)
  - drops rapidly to zero in spacer region due to current spreading
- h_e
  - slightly dependent on y due to voltage drop across base resistance components



$\Rightarrow$ h_J determines the lateral distribution of h

$\Rightarrow$ hole charge contribution only from region with (vertical) current flow

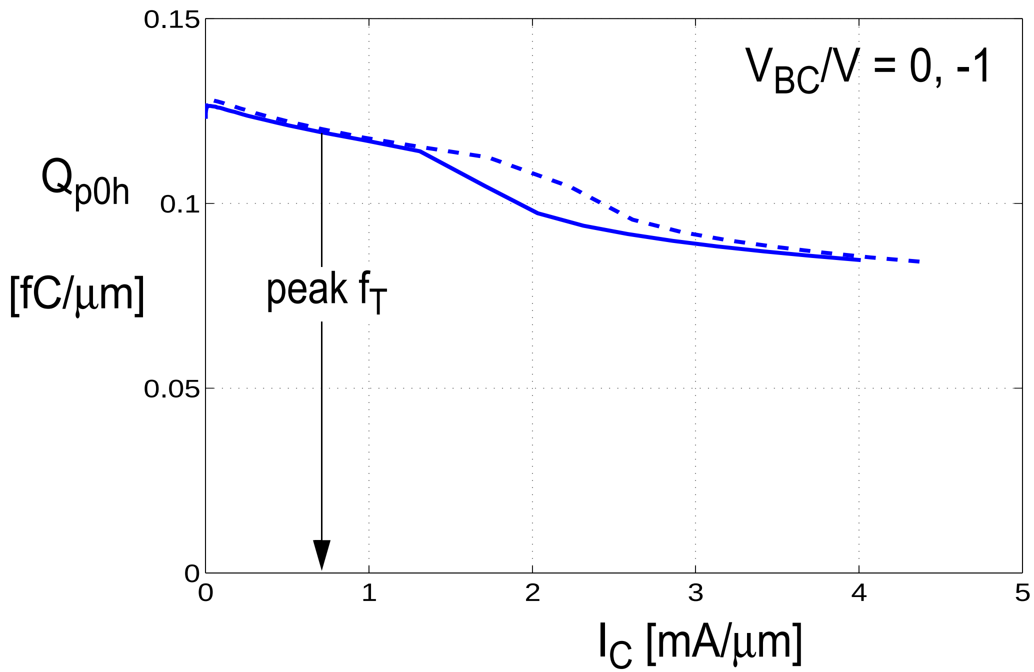
Theory (cont'd)

evaluation of hole density related integrals

- split p into zero-bias and bias dependent component: $p = p_0 + \Delta p$
- zero-bias weighted hole charge

$$Q_{p0h} = q2l_{E0}\int_0^{b_{bc}}\int_0^{x_{C'}} hp_0 dxdy$$

  - slightly bias dependent due to "shape" change of h components



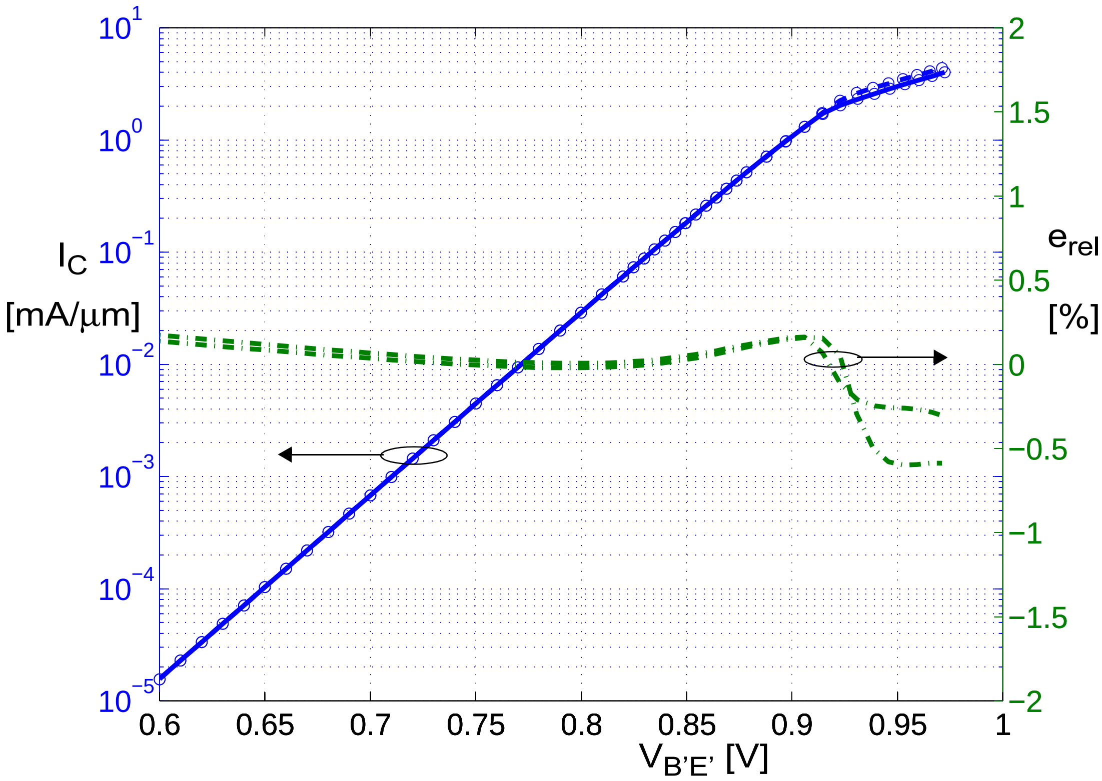
- bias dependent weigthed hole charge

$$\Delta Q_{ph} = q2l_{E0}\int_0^{b_{bc}}\int_0^{x_{C'}} h\ \Delta p\ dxdy$$

Master equation

$$I_T = q b_E l_{E0} c_0 \frac{\exp\left(\frac{V_{B'E'}}{V_T}\right) - \exp\left(\frac{V_{B'C'}}{V_T}\right)\frac{2b_{bc}}{b_E}}{Q_{p0h} + \Delta Q_{ph}}$$

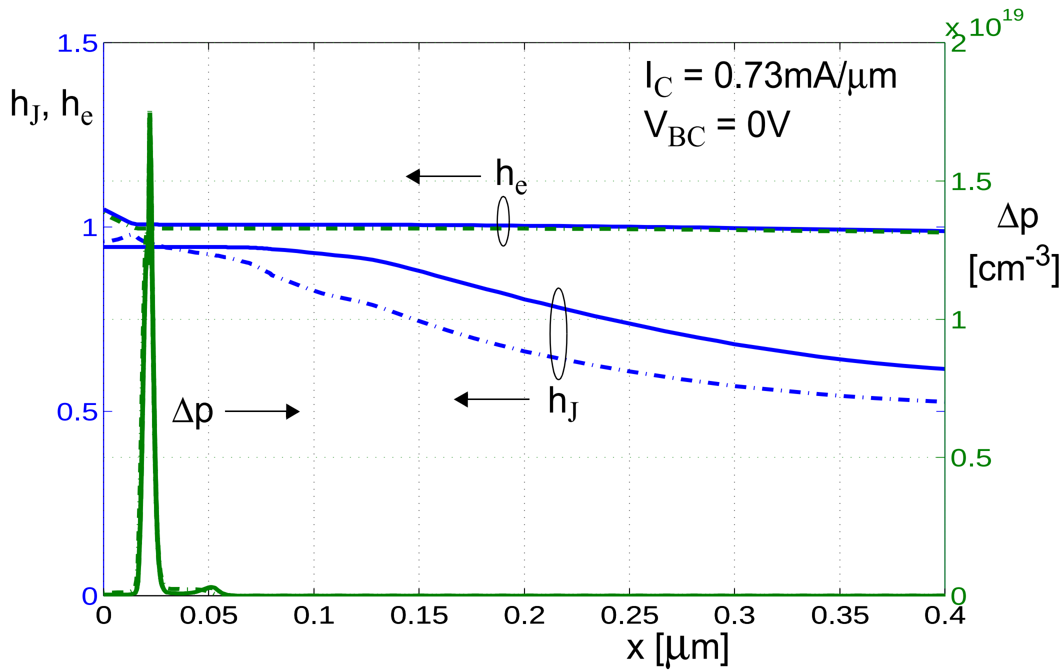
- evaluation with numerical values for charges and h
- excellent agreement over whole bias region
- slight deviation (0.5%) from device simulation due to numerical integration



⇒ suitable for compact model development

Model equations

- need to represent ΔQ_{ph} by measurable expression, such as ΔQ_p components
- partition ΔQ_p into space-charge and minority components: $\Delta Q_p = Q_{jE} + Q_{jC} + Q_m$
- split weighting function into "mostly"
  - vertical component $h_g = h_g(x,0)$ $\Rightarrow$ treat like 1D case
  - lateral component $h_2 = h_J h_e = h_2(x_{bm},y)$

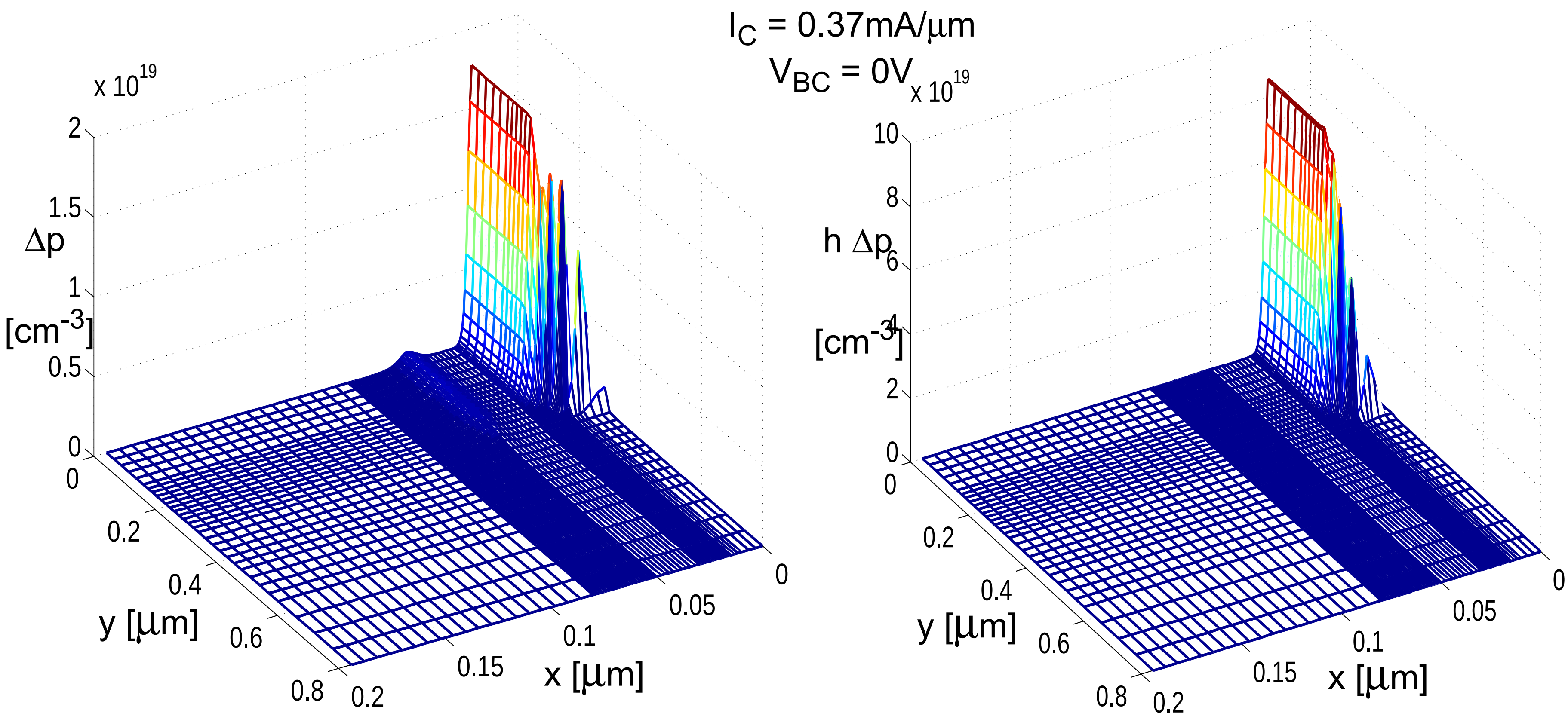


solid lines: $y = 0$

dashed lines: $y = b_{E0}/2$

Appendix: Theory (cont'd)

3D distribution of Δp and h Δp in the BE junction region



$\Rightarrow$ suppression of part of the *perimeter* BE depletion charge by h (h_J)

Model equations (cont'd)

- example:

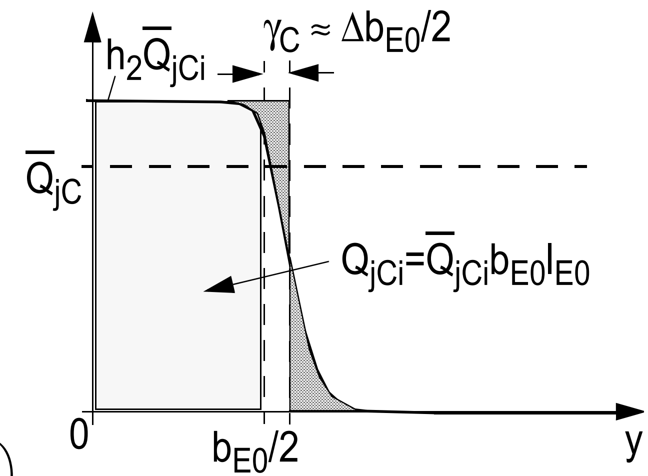
BC space charge $Q_{jCh} = 2l_{E0}\int_0^{b_{bc}} h_2\left(q\int_{w_{bc}} h_g \Delta p dx\right)dy \cong 2l_{E0}\bar{h}_{gjC}\int_0^{b_{bc}} h_2 \bar{Q}_{jC} dy = \bar{h}_{jC} Q_{jC}$

with

- Q_{jC} as total (internal and external) BC space charge measurable at the terminals
- $\bar{Q}_{jC}$ as charge per area (equals under emitter window the *internal* charge density $\bar{Q}_{jCi}$)
- $\bar{h}_{jC} = \bar{h}_{gjC}\bar{h}_2$ with bias independent $\bar{h}_2$ as first-order approximation
- at low injection:

$$Q_{jCh} = \bar{h}_{jC} Q_{jC} \approx \bar{h}_{gjC} Q_{jCi}\left(1 + \frac{2}{b_{E0}}\int_{b_{E0}/2}^{b_{bc}} h_2 dy\right)$$

$$\approx \bar{h}_{gjC} Q_{jCi}\left(1 + \frac{\Delta b_{E0}}{b_{E0}}\right)$$

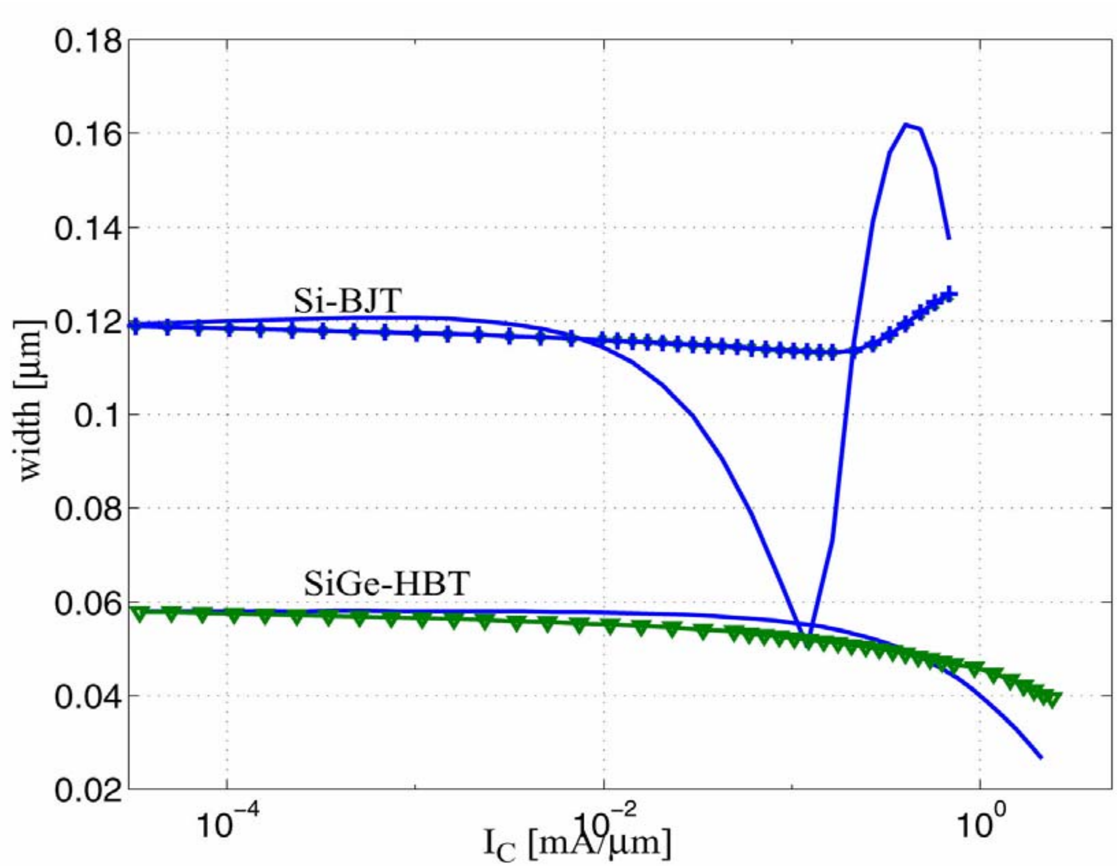


- similarly:

$$Q_{p0h} \approx \bar{h}_{g0} Q_{p0i}\left(1 + \frac{\Delta b_{E0}}{b_{E0}}\right), \quad Q_{jEh} = \bar{h}_{gjE} Q_{jEi}\left(1 + \frac{\Delta b_{E0}}{b_{E0}}\right)$$

Model equations (cont'd)

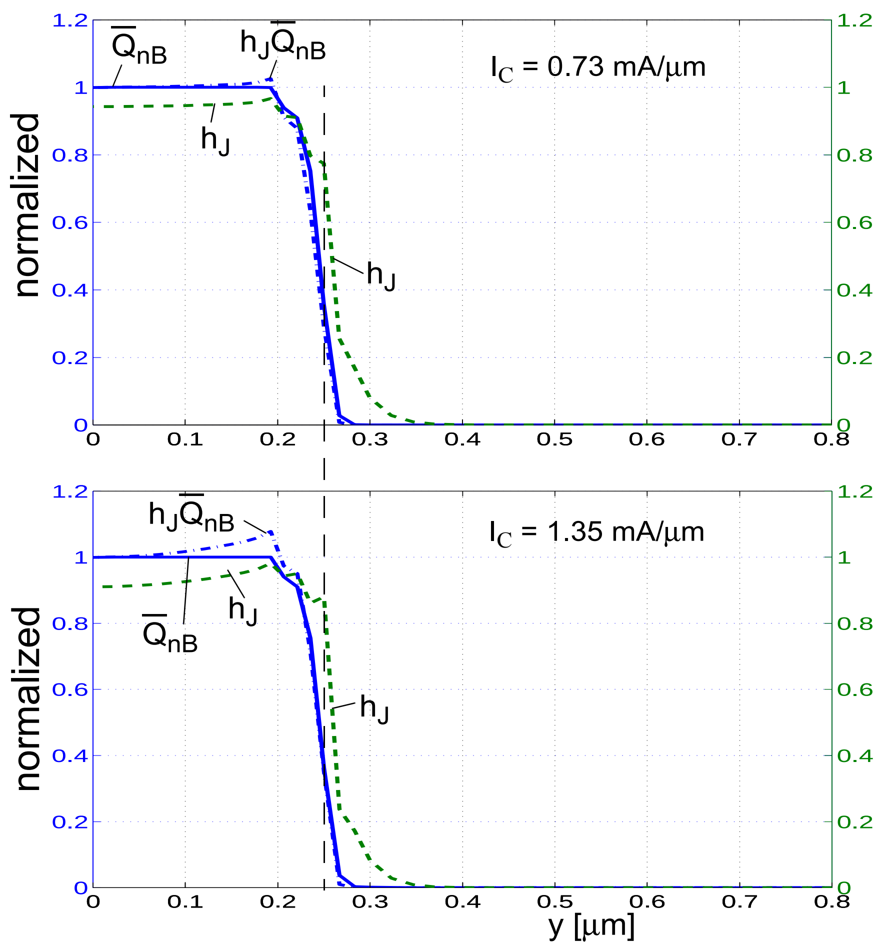
comparison between $\Delta b_{E0}/2$ and γ_C



- very close at low current densities
- deviations at higher current densities caused by different way of accounting for r_B

Model equations (cont'd)

minority charge



- base region: $Q_m = Q_{nB}$
- internal electron charge density:
  $\overline{Q}_{nBi} = \overline{Q}_{nB}(y=0) \Rightarrow Q_{nBi} \approx \overline{Q}_{nBi} b_{E0} l_{E0}$
- normalized electron charge density
  $\overline{Q}_{nB}(y)/\overline{Q}_{nBi} \propto h_2(y)$

$$\Rightarrow h_2\,(\overline{Q}_{nB}(y)/\overline{Q}_{nBi}) \propto h_2^2(y)$$

$$\Rightarrow \int_0^{b_{bc}} h_2 \overline{Q}_{nB}\, dy \approx \overline{Q}_{nBi} \frac{b_{E0}}{2}$$

- first-order approximation:

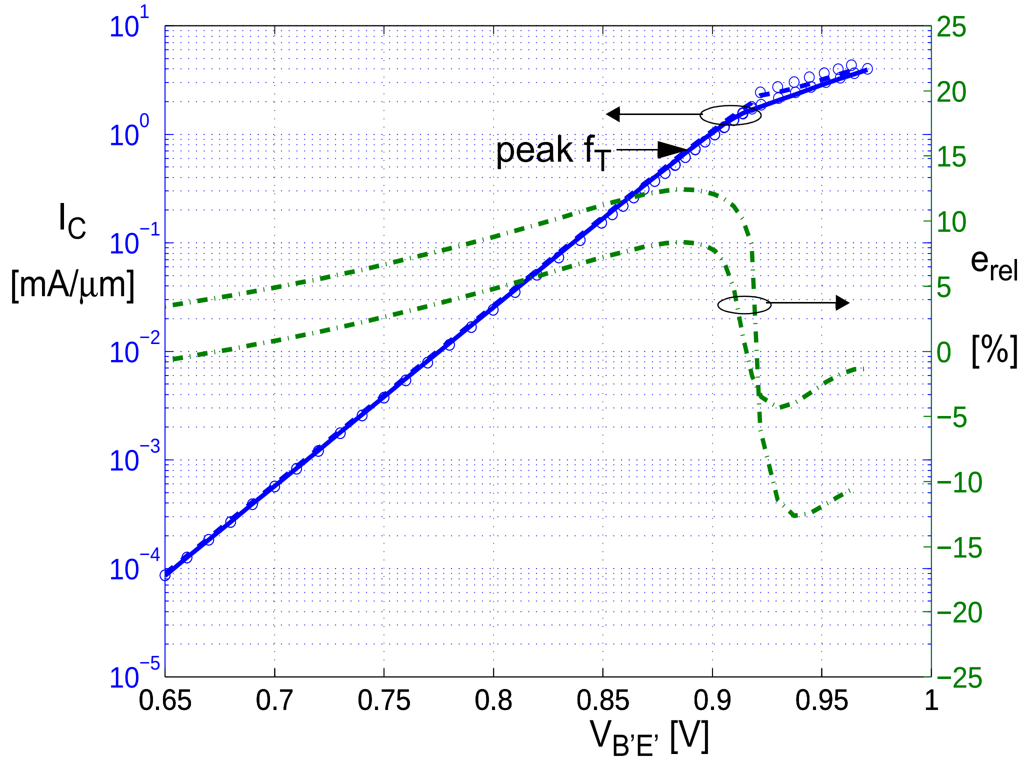
$$Q_{mh} \approx \bar{h}_{mE} Q_{pE} + \bar{h}_{mB} Q_{nB} + \bar{h}_{mC} Q_{pC}$$

with $\overline{h}_m$ as ***spatial*** and ***bias*** averages

Model equations: comparison to reference

- model equation: $I_T = qb_E l_{E0} c_0 \dfrac{\exp\left(\dfrac{V_{B'E'}}{V_T}\right) - \exp\left(\dfrac{V_{B'C'}}{V_T}\right)\dfrac{2b_{bc}}{b_E}}{Q_{p0h} + \bar{h}_{jE}Q_{jE} + \bar{h}_{jC}Q_{jC} + Q_{mh}}$

- numerical values for charges
- (bias) averaged weighting factors
- good agreement in relevant bias region
  - SiGe HBTs: deviation caused mostly by change of h_J*h_e "form factor"
- improvements:
  - make h_x bias dependent
  - normalize to $\bar{h}_{mB}$ to reduce parameter number and simplify modeling of weighting factors



$\Rightarrow$ existing compact models all use $\bar{h}_x$

Conclusions

- The derivation of a multi-dimensional (generalized) Integral Charge-Control Relation for the transfer current of BJTs/HBTs has been presented
- The resulting "master equation" has been verified by device simulation
- The "master equation" can serve for deriving simplified formulations for compact models
  - keep control over errors
  - well-defined evaluation of the impact of assumptions
  - clear meaning of variables and parameters (particularly as function of lateral dimensions)
- A first-order approximation of the master equation has been derived
  - equivalent to formulation used in existing model
  - demonstration of achievable accuracy

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