About Modeling the Reverse Early Effect in HICUM Level 0

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Purpose

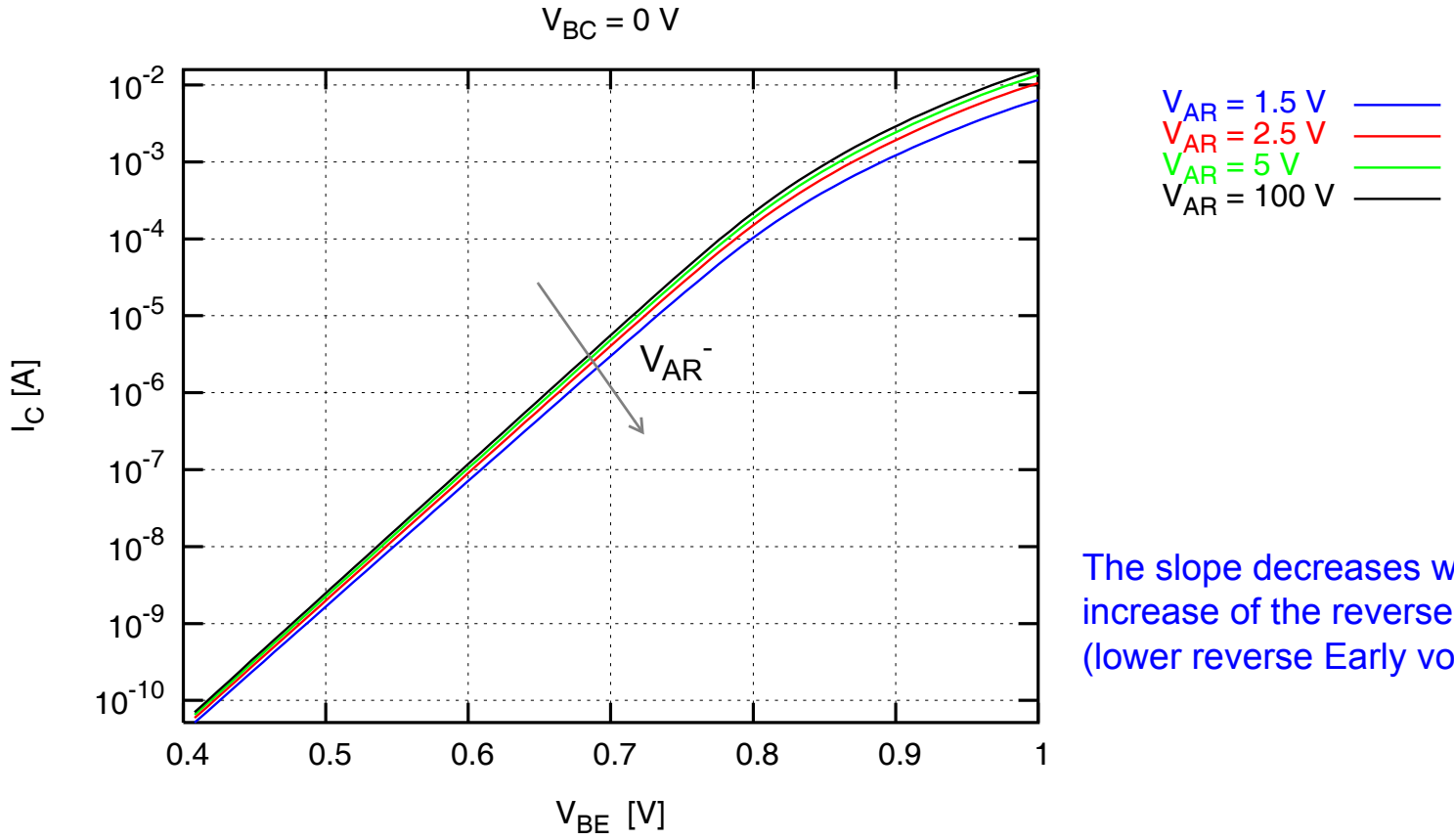
- According to the bipolar models, the reverse Early effect is modeled in various ways
  - From device physics in models such as HICUM, MEXTRAN or VBIC.
  - With more or less important approximations for models like SPICE Gummel-Poon (SGP) or HICUM Level 0.
- The question is to know if these approximations are justified and which are their impacts on the model accuracy.
- Application to HICUM/L0

Outline

- Introduction
- Why Reverse Early effect is so important?
- How is it implemented in BJT models?
- Experimental results and HICUM/L0 limitation
- Summary

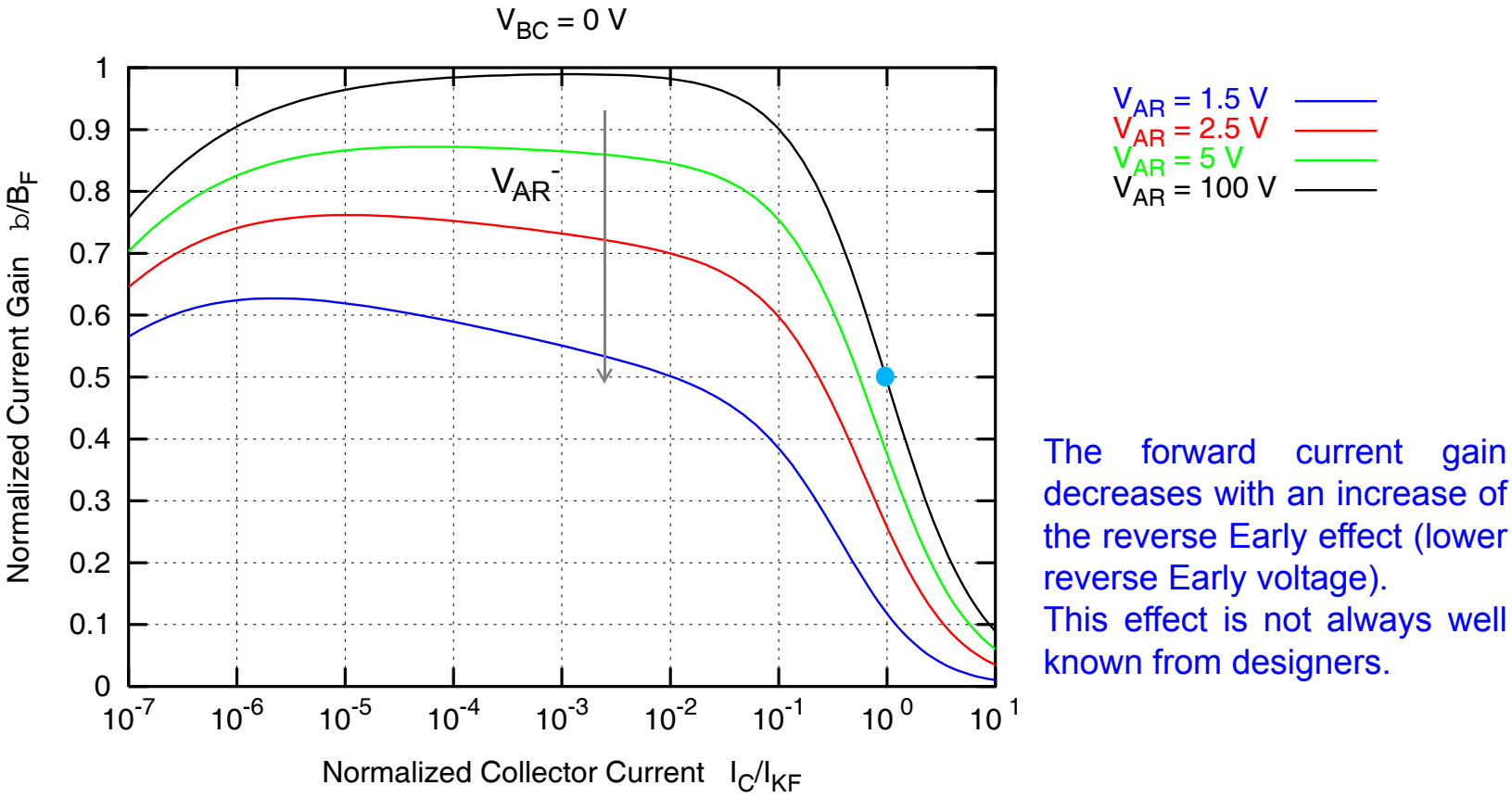
Introduction

- Despite its name, the *reverse* Early effect affects strongly the collector current in *forward* mode.
- Difficult to observe on forward Gummel plots, its effect being concealed by the exponential dependence of I_C with V_{BE}



The slope decreases with an increase of the reverse Early effect (lower reverse Early voltage)

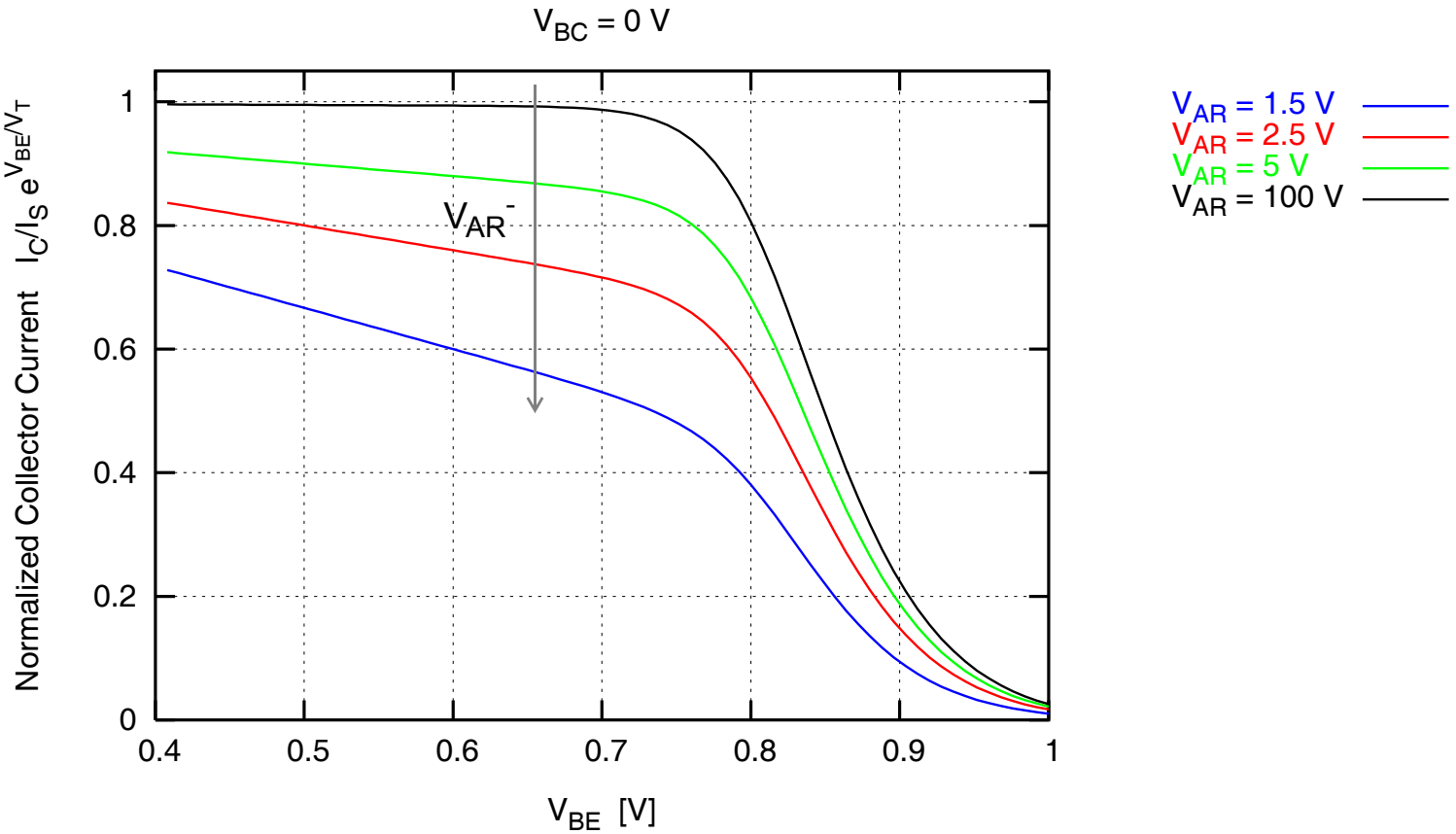
➔ But clearly visible on the forward current gain...



The forward current gain decreases with an increase of the reverse Early effect (lower reverse Early voltage).
This effect is not always well known from designers.

- We can observe that the forward current gain is divided by 2 at $I_C = I_{KF}$. It is a way to directly extract the forward knee current I_{KF} of the SGP model, from the forward current gain corrected of the reverse Early effect.

➔ ...or better on the *normalized* collector current $I_C/I_S e^{V_{BE}/V_T}$



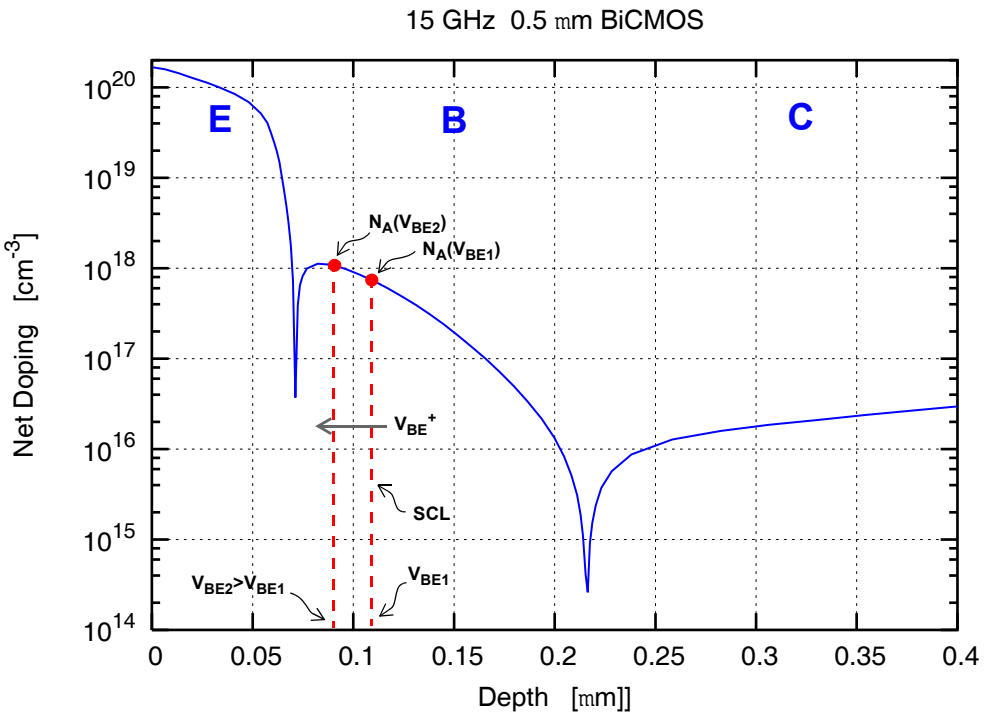
- We can notice, that in fact this *normalized* collector current is, at low and medium V_{BE} (linear part of the characteristic), simply the inverse of the normalized hole charge Q_p/Q_{p0}.
- The reverse Early effect can be directly quantified from the slope of this *normalized* collector current.
- It is the only way to directly extract (linear regression), without any ambiguity, the reverse Early voltage **V_{AR}** of the SGP model.

Why the Reverse Early Effect so Important?

- Whatever the technology, for vertical (conventional) BJTs, the reverse Early effect is not negligible (low equivalent reverse Early voltage on the order of 1 to 5V). Why?
- Initially, the inverse Early voltage V_{AR} was introduced, in the SGP model, in order to take into account the base width modulation with V_{BE}.
- In fact, in advance bipolar technologies, the base doping is very high, and consequently, the base width is less affected by a V_{BE} variation.
- Therefore, low reverse Early voltages, for this kind of technologies, can not be explained only by a modulation of the base resulting from a variation of the BE space-charge layer width.
- 2 possible explanations
  - The reverse Early voltage does not depend only on the base concentration, but also on the shape of the base doping (Si BJTs).
  - Reverse (effective) Early voltage can be also effected by the Ge profile in the base of HBTs.

Effect of the Shape of the Base Doping on $I_C(V_{BE})$

➔ Si BJTs



- The collector current depends on the injected electron in the base, at the EB depletion layer boundary

$$I_C \propto n_{b0}(V_{BE}) \cdot e^{\frac{V_{BE}}{V_T}} \quad \text{or} \quad I_{C^*} = \frac{I_C}{e^{V_{BE}/V_T}} \propto n_{b0}(V_{BE})$$

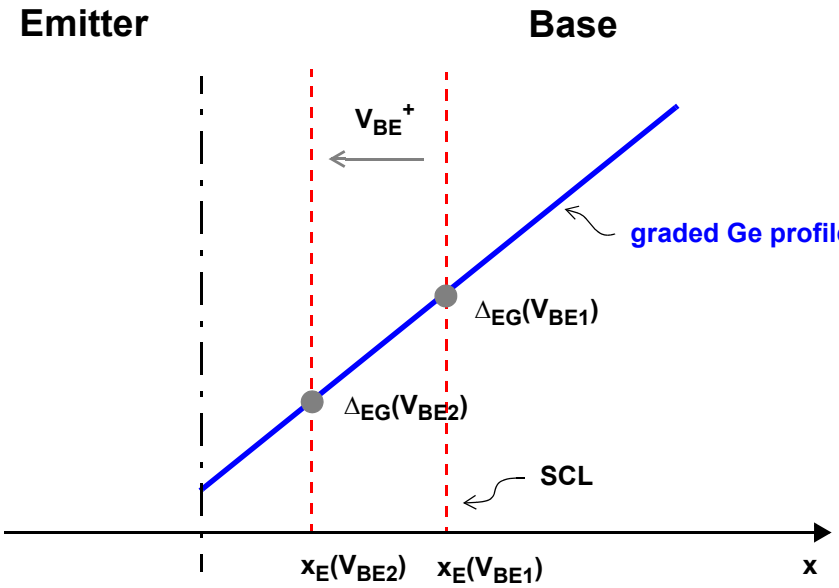
- If V_{BE} increases $N_A(V_{BE2}) > N_A(V_{BE1})$
- Since $n_{b0} \approx \frac{n_i^2}{N_A}$, we have $n_{b0}(V_{BE2}) < n_{b0}(V_{BE1})$

  and therefore $\boxed{I_{C^*}(V_{BE2}) < I_{C^*}(V_{BE1})}$

- Similar effect than the reverse Early effect (base width modulation with V_{BE}), where I_{C^*} decreases with V_{BE}.

Effect of the Ge Base Profile on the $I_C(V_{BE})$

➔ HBTs



- In HBTs, the collector current depends on the reduction Δ_{EG} of the base bandgap due to the introduction of Ge into the base

$$I_{C^*} = \frac{I_C}{e^{V_{BE}/v_T}} \propto e^{\frac{\Delta_{EG}}{v_T}}$$

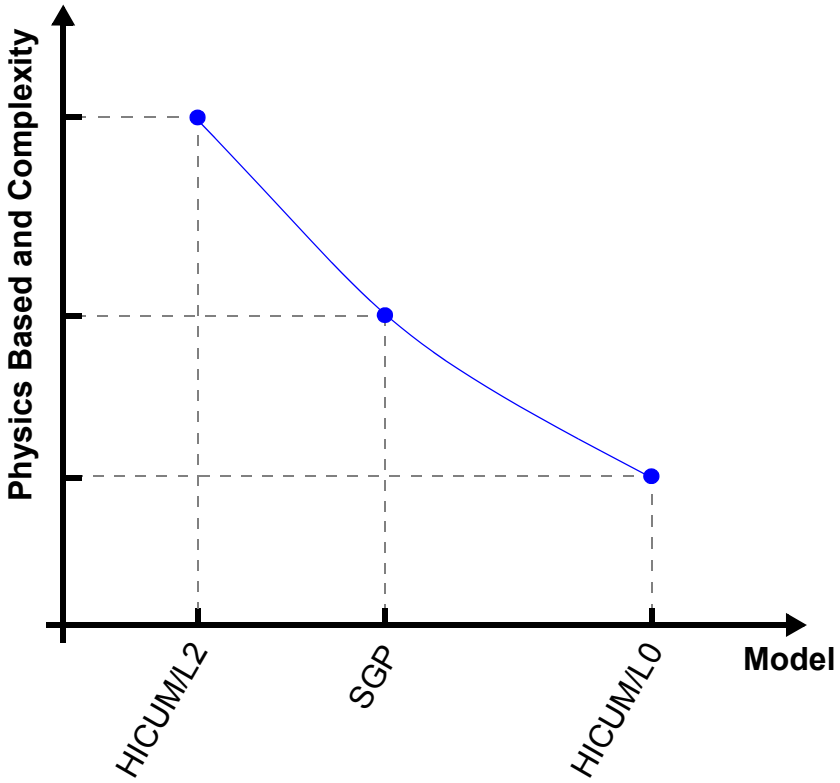
- In graded Ge base profile (triangle or trapezoidal), if V_{BE} increases $\Delta_{EG}(V_{BE2}) < \Delta_{EG}(V_{BE1})$

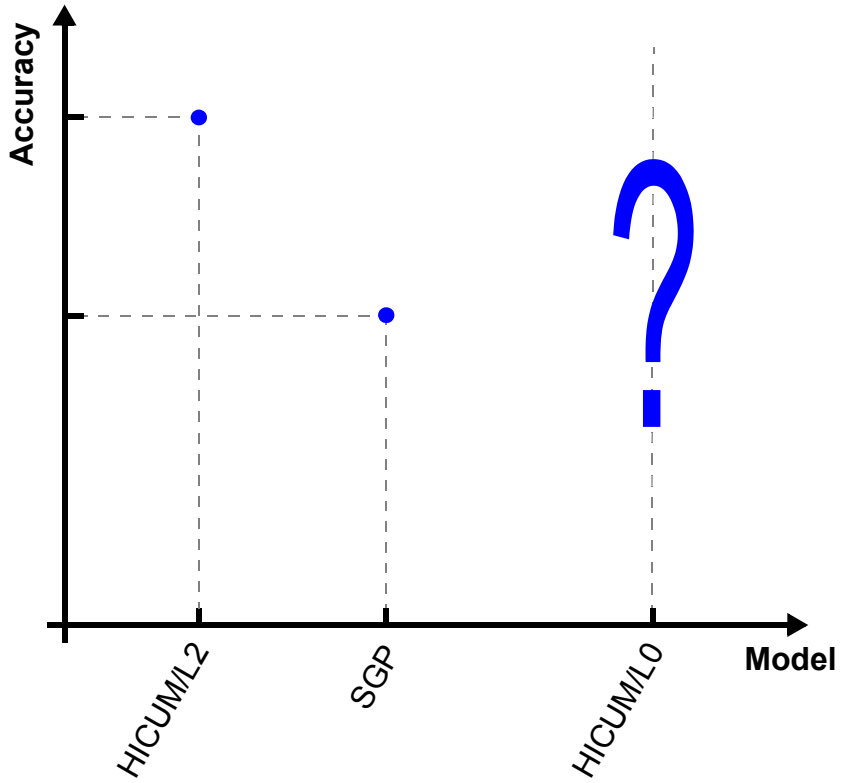
  and therefore $I_{C^*}(V_{BE2}) < I_{C^*}(V_{BE1})$

- This effect is responsible of the very low *effective* reverse Early voltage (decrease of I_{C^*} with V_{BE}) of S_iG_e HBTs with graded Ge base profile.

How the Reverse Early Effect is Implemented in BJT Models?

➔ Depending on the model, several approaches more or less physics based





➔ HICUM Level 2

- Low injection, $V_{BC} = 0$ V

$$I_C \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{1 + h_{jEi} \cdot \frac{Q_{jEi}}{Q_{p0}}} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{1 + \mathbf{h_{jEi}} \cdot \mathbf{q_{jEi}}}$$

$\mathbf{h_{jEi}}$ weighting factor in HBTs

$\mathbf{q_{jEi}}$ normalized BE depletion charge

➔ SPICE Gummel-Poon

- Low injection, V_{BC} = 0 V

$$q_{jEi} = \frac{1}{Q_{p0}} \cdot \int_0^{V_{BEi}} C_{jEi}(V)\ dV$$

In SGP model, C_{jEi} is assumed to be constant and equal to its average value $\overline{C_{jEi}}$

$$I_C \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{1 + \mathbf{h_{jEi} \cdot q_{jEi}}} \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{1 + \frac{h_{jEi} \cdot \mathbf{\overline{C_{jEi}}} \cdot V_{BEi}}{\mathbf{Q_{p0}}}} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{1 + \frac{V_{BEi}}{\mathbf{V_{AR}}}} \quad \text{with} \quad \mathbf{V_{AR}} = \frac{\mathbf{Q_{p0}}}{\mathbf{h_{jEi} \cdot \overline{C_{jEi}}}}$$

- Moreover, In SGP model it is assumed (questionnable for advanced HBTs) $\mathbf{V_{AR}}$ >> V_{BEi}

$$1 \Big/ \left\{1 + \frac{V_{BEi}}{\mathbf{V_{AR}}}\right\} \approx 1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}, \text{ and finally } \boxed{I_C \approx \left\{1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}\right\} \cdot I_S \cdot e^{\frac{V_{BEi}}{V_T}}}$$

➔ HICUM Level 0

- Low injection, V_{BC} = 0 V

  Same approximation than in SGP model, it is assumed that $\mathbf{V_{AR}} >> V_{BEi}$.

  Therefore $1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}$ can be considered as the first two terms of the Taylor series expansion of $e^{-\frac{V_{BEi}}{\mathbf{V_{AR}}}}$

$$I_C \approx \left\{1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}\right\} \cdot I_S \cdot e^{\frac{V_{BEi}}{V_T}} \approx I_S \cdot e^{-\frac{V_{BEi}}{\mathbf{V_{AR}}}} \cdot e^{\frac{V_{BEi}}{V_T}} = I_S \cdot e^{\frac{V_{BEi}}{V_T} \cdot \left\{1 - \frac{V_T}{\mathbf{V_{AR}}}\right\}}$$

We can consider $V_T/\mathbf{V_{AR}} << 1$, $1 - \frac{V_T}{\mathbf{V_{AR}}}$ can be rewritten as $1 - \frac{V_T}{\mathbf{V_{AR}}} \approx 1 / \left\{1 + \frac{V_T}{\mathbf{V_{AR}}}\right\}$,

which gives the final expression of I_C

$$I_C \approx I_S \cdot e^{\frac{V_{BEi}}{\left\{1 + \frac{V_T}{\mathbf{V_{AR}}}\right\} \cdot V_T}} = I_S \cdot e^{\frac{V_{BEi}}{\mathbf{M_{CF}} \cdot V_T}}$$

with $\mathbf{M_{CF}} = 1 + \frac{V_T}{\mathbf{V_{AR}}}$

➔ Few comments concerning the relation between M_{CF} and V_{AR}

- As $M_{CF} = 1 + \frac{V_T}{V_{AR}}$ and $V_{AR} >> V_T$

  therefore M_{CF} is very close to 1. In order to reproduce accurately the reverse Early effect, M_{CF} must be given at least with **5 digits**.

V_{AR} (V)	M_{CF}
1.5	**1.01724279**
2.5	**1.01034567**
5	**1.00517284**
100	**1.00025864**

M_{CF} (4 digits)	V_{AR} (V)
1.017	1.5**21**
1.010	2.5**86**
1.005	5.**173**
1.000	∞

M_{CF} (5 digits)	V_{AR} (V)
1.0172	1.5**04**
1.0103	2.5**11**
1.0051	5.0**71**
1.0002	**129.32**

- As $M_{CF} = 1 + \frac{V_T}{V_{AR}}$, it *could be* temperature dependant (assuming V_{AR} temperature independent).

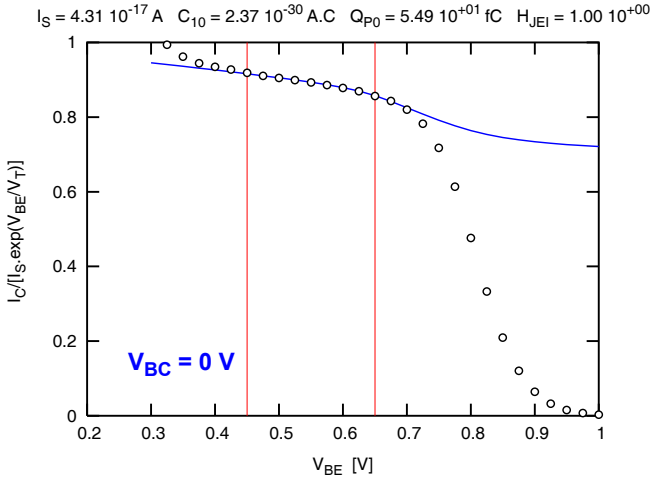
  To be investigated in more detail (to a next HICUM workshop!...)

Temperature (°C)	V_{AR} (V)	M_{CF}
-50	2.5	**1.00769**
27	2.5	**1.01034**
150	2.5	**1.01458**

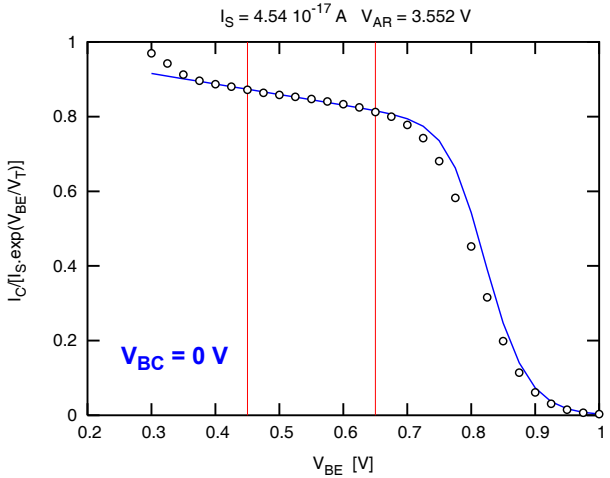
Experimental Results

➔ HICUM/L2, SGP, HICUM/L0 [1]

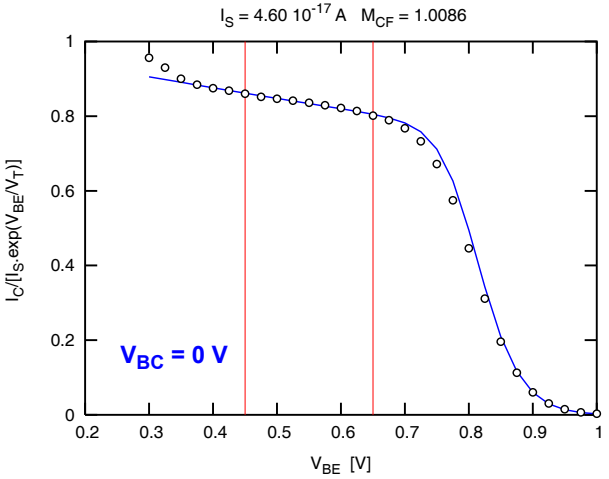
- High current parameters not fully extracted



HICUM/L2



SGP

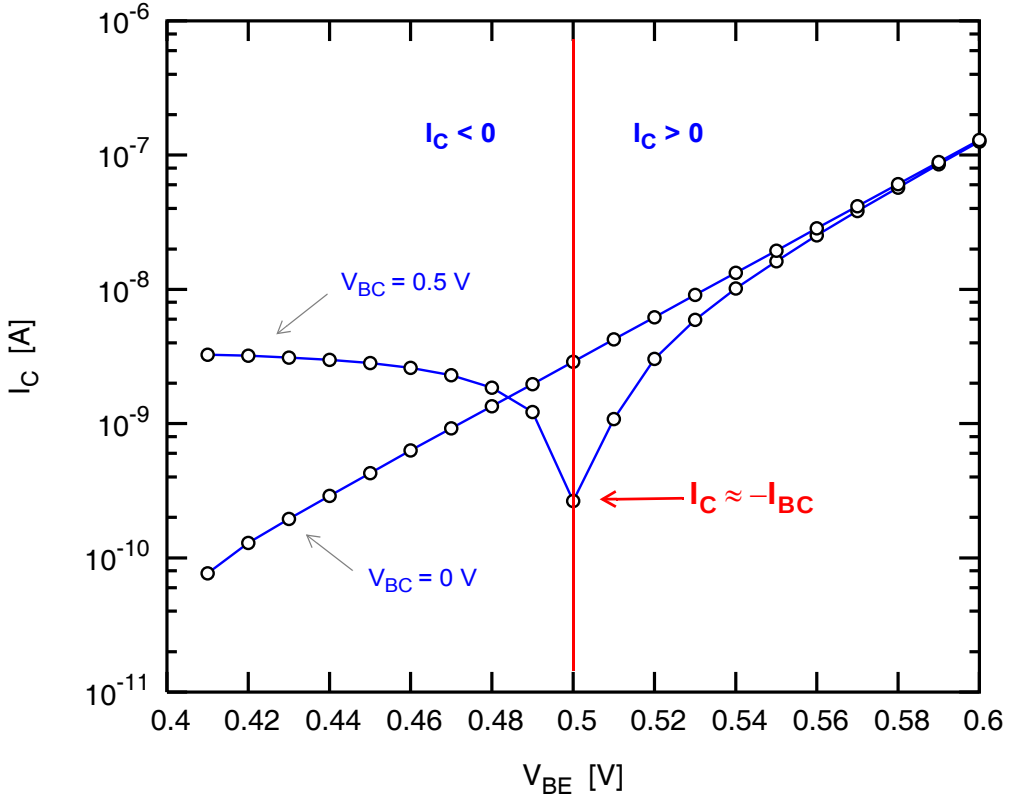


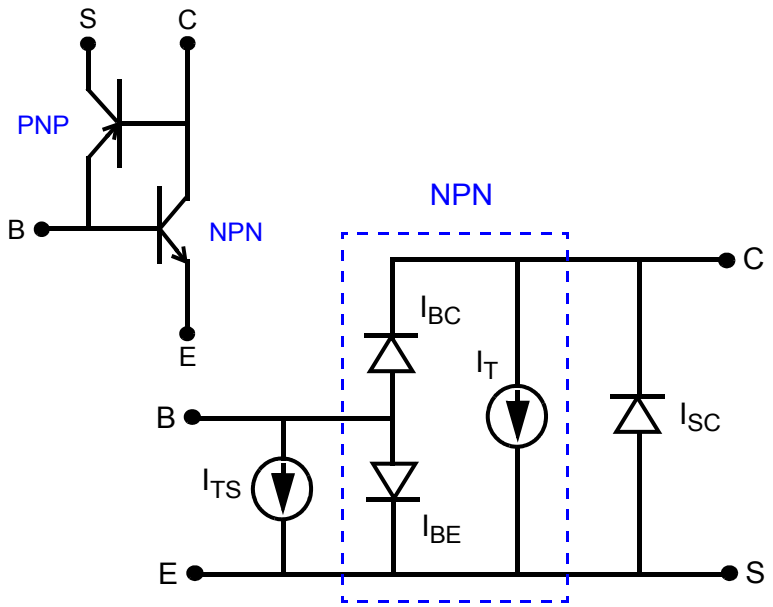
HICUM/L0

- Despite the different approaches used to model the reverse Early effect, all models give similar accuracy (negative slope well described) on the normalized collector current (or current gain), at V_{BC} = 0 V.
- But it is no more valid for positive V_{BC}, in the saturated region.

1. ST solver

Gummel PLots with Positive V_{BC}





DC low current simplified equivalent circuit

- $I_C = I_T - I_{BC} - I_{SC}$

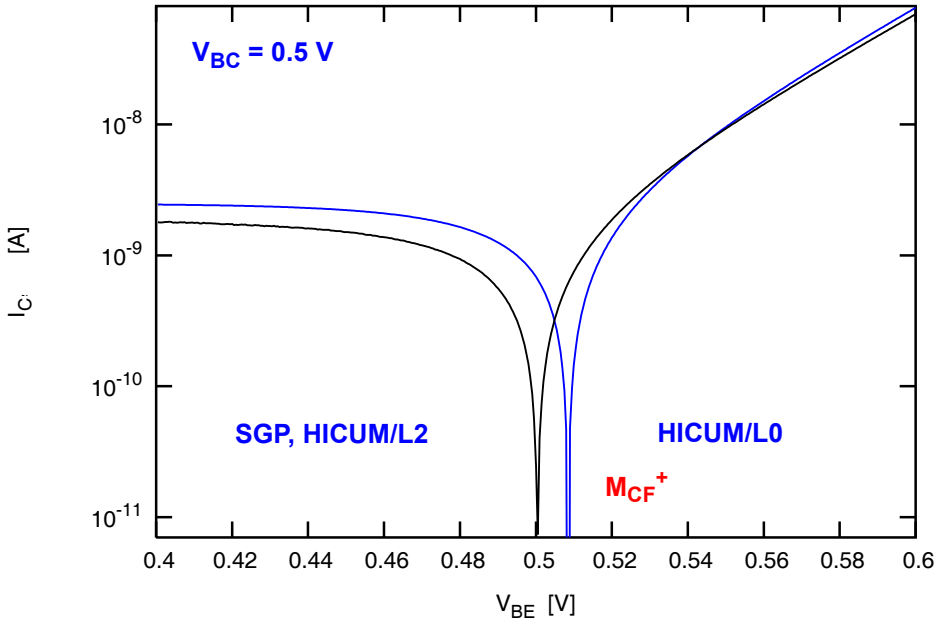
- at $V_{BC} = V_{BE}$ (0.5 V)

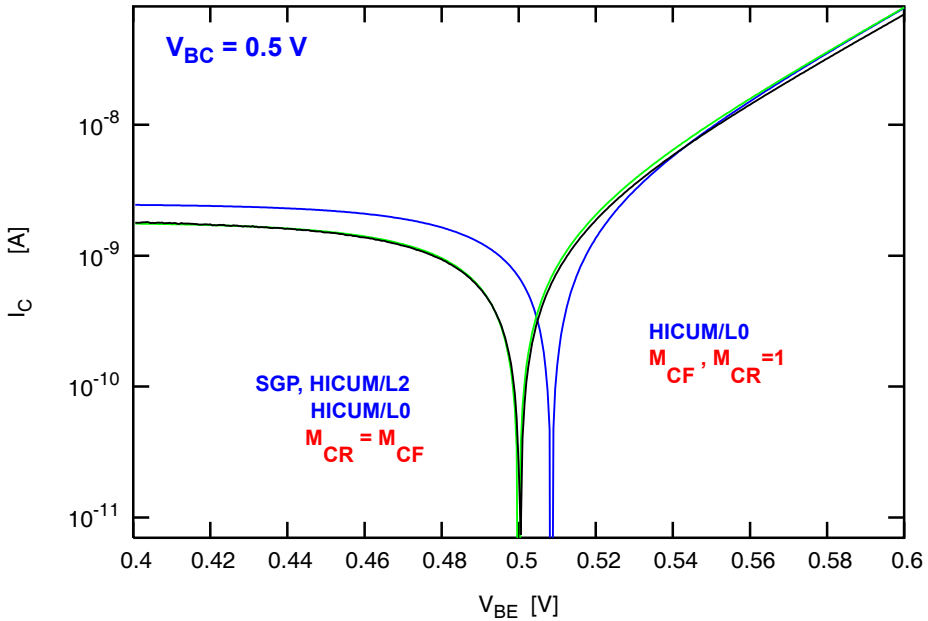
$$\begin{cases} I_{SC} \approx 0 \quad \text{because Vsc = 0} \\ I_T \propto e^{\frac{V_{BE}}{V_T}} - e^{\frac{V_{BC}}{V_T}} = 0 \\ I_C \approx -I_{BC} \end{cases}$$

but not in HICUM/L0

$$I_T \propto e^{\frac{V_{BE}}{M_{CF} \cdot V_T}} - e^{\frac{V_{BC}}{M_{CR} \cdot V_T}} \neq 0$$

➔ Limitation of HICUM/L0 and solution to minimize this modeling issue



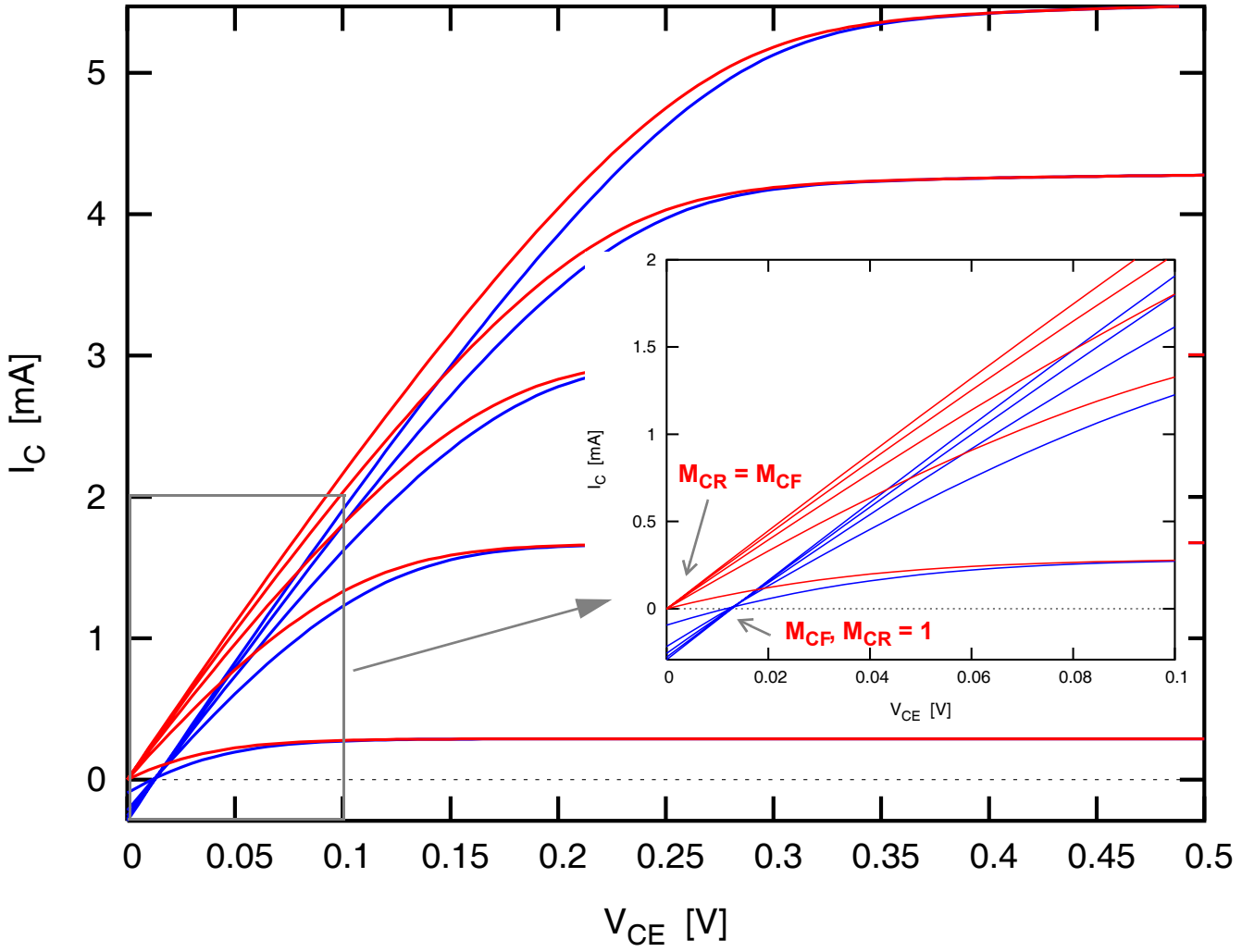


- In saturated region, and especially for $V_{BE} = V_{BC}$, the shift between HICUM/L0 and other models increases with M_{CF} (high reverse Early effect, low reverse Early voltage).
- To minimize this effect, a solution consists of set **M_{CR}** equal to **M_{CF}**. Therefore, in this case, the transfer current I_T is equal to zero at $V_{BE} = V_{BC}$

$$I_T \propto e^{\frac{V_{BE}}{M_{CF} \cdot V_T}} - e^{\frac{V_{BC}}{M_{CR} \cdot V_T}} = 0 \quad \text{if } M_{CF} = M_{CR}$$

- This correction is to the detriment of the accuracy on the reverse Gummel plot (slope to low).

➔ The same behavior can be observed on the forward output characteristics. Here again, M_{CR} needs to be set to M_{CF} in order to fit accurately the characteristics at low V_{CE}.

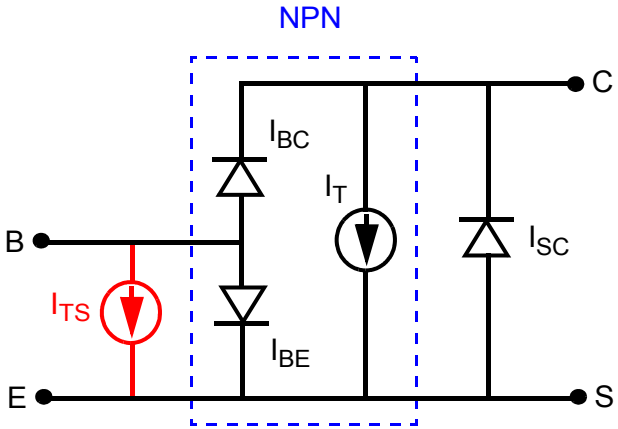


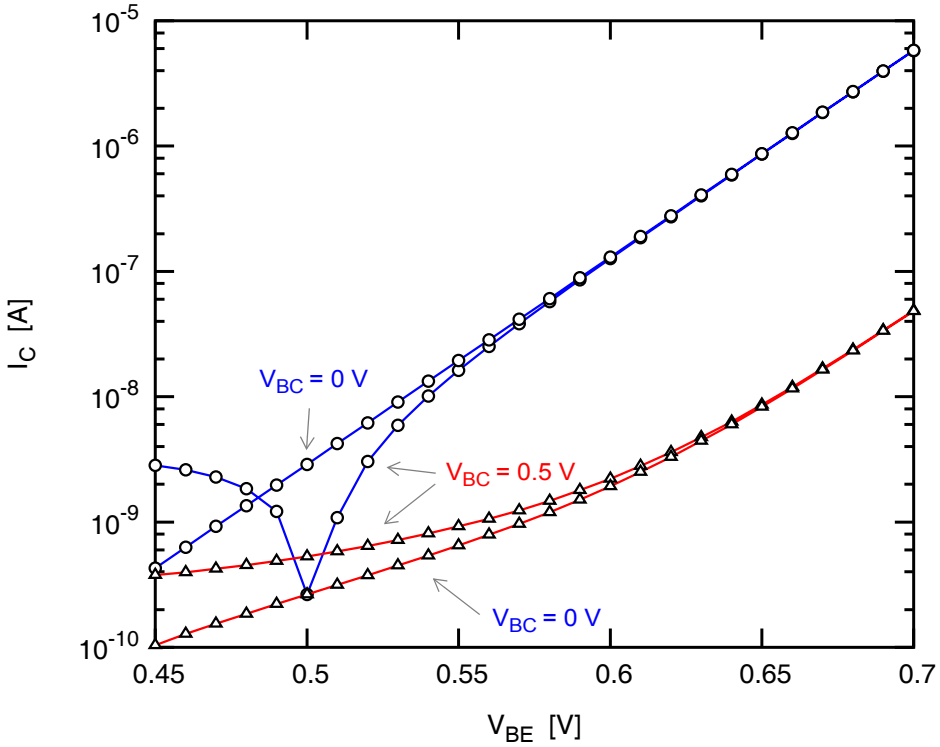
Summary

- The modeling of the reverse Early effect in HICUM/L0 has been investigated.
- Its formulation is a simplification of the SPG model which leads to the introduction of a non-ideality factor M_{CF} on the forward transfer current.
- We have demonstrated that in some bias conditions (saturated region), this model can be not enough accurate.
- To overcome this model issue, 2 possibilities:
  - To use the same non-ideality factor in forward and reverse ($M_{CR} = M_{CF}$). Inconsistent with a correct description of the reverse Gummel characteristics.
  - To come back to the SGP model formulation by using a reverse Early voltage V_{AR} instead of a non-ideality factor M_{CF}.
- One pending question: as M_{CF} is linked to V_{AR} via V_T, can we assume M_{CF} temperature independent?

Addenda (1/2)

- An other important limitation of HICUM/L0, in the saturated region, is the lack of parasitic substrate PNP.
- With the present model it is not possible
  - To describe the bias dependence of the forward Gummel plot (especially $I_{B)}$ with positive V_{BC}.
  - $I_C = I_T - I_{BC} - I_{SC}$
  - $I_B = \mathbf{I_{TS}} + I_{BE} + I_{BC}$





- To accurately model the output characteristics at low V_{CE}.

Addenda (2/2)

➔ Therefore, the implementation of the parasitic substrate PNP, in HICUM/L0 is always requested (using a flag if needed).