

Solving the Heat Equation with the Quadrupole Approach

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Outline

- Introduction and definition
- Theoretical Background
 - R: steady state
 - C: transient state
 - Cascading layers
 - General form
 - The thermal impedance
- Applications: SiGe HBT
 - Scaling issues
 - The thermal capacitance
 - Flip chip packaging
 - The toolkit
- Conclusion

Modeling of diffusive heat transfer, SOA

- Main numerical tools
 - Finite differences, Finite elements, Finite volumes, Boundary elements
 - Complex geometry, Non-linear problem
- Analytical modeling tools
 - Simple geometry (1D), steady state
 - Method of separation of variables (time-space series)
 - Method of Green's functions → solution in an integral form for the temperature field
 - “However these methods are rather painful to implement in practical situations and simulation of heat transfer within a multimaterial such as multilayer composite medium becomes a nightmare” from D. Maillet et al, Thermal Quadrupoles, Wiley & sons 2000

Modeling of diffusive heat transfer, Quadrupole method

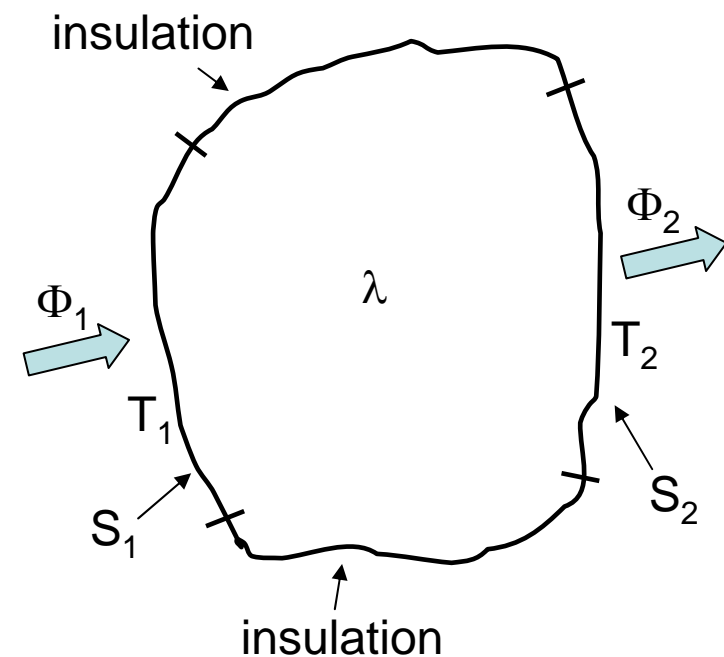
- Instead of looking for a solution that is temperature in the time-space domain, seek a solution in a transformed domain.
- Laplace transform of the time variable
 - Laplace temperature: very simple explicit form (very often)
 - Linear matrix equation linking the input vector (Laplace temperature, Laplace heat flux) to the corresponding output vector
- Approach presented by H.S. Carslaw, 1921

Definition of the quadrupole method

- Unified exact explicit method of representation of linear systems
- Based on 2x2 matrices that relate some *transform* of both temperature and flux on one surface of a considered medium to the same quantities on another surface
- Transform means:
 - Laplace transform
 - Fourier space cosine transform in steady state case
 - Combination of preceding transforms
- It allows calculation of temperature and flux vector
- It brings a *representation* of heat transfer through multimaterials by networks of *impedances*

Case a: Steady-State Transfer

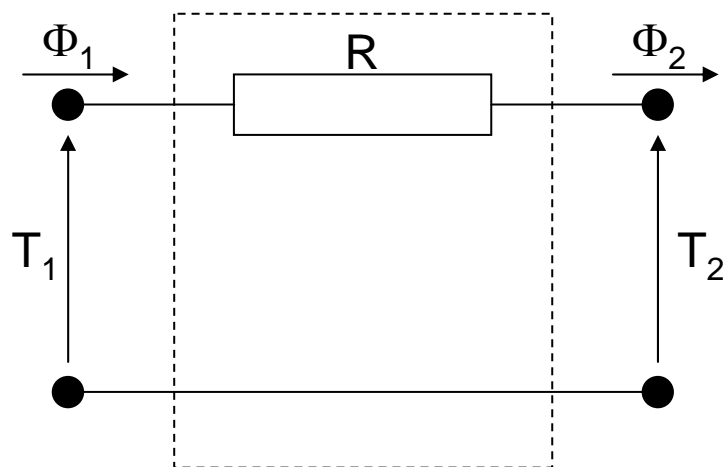
- Single homogenous material, limited by a closed surface S
- Heat input $\Phi_1 \Rightarrow S_1$ with T_1 , heat output $\Phi_2 \Rightarrow S_2$ with T_2 , remaining S : insulated
- Steady state:
 - $T_1 - T_2 = R \Phi_1$
 - $\Phi_1 = \Phi_2$



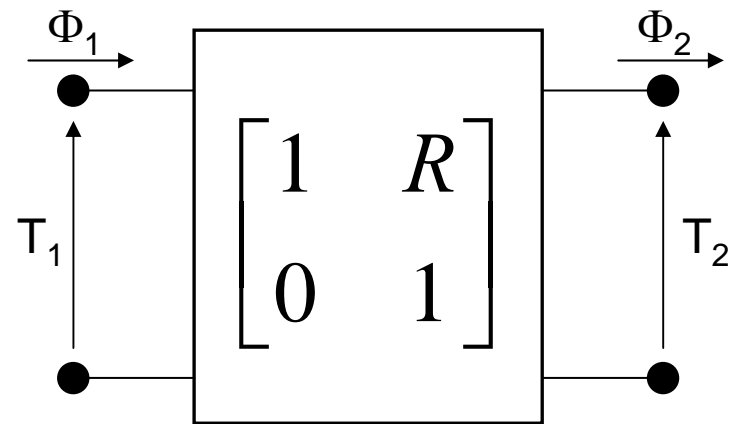
Case a: Matrix and Network representation

- Matrix equation form:
– $T_1 - T_2 = R \Phi_1$
– $\Phi_1 = \Phi_2$
$$\begin{bmatrix} T_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_2 \\ \Phi_2 \end{bmatrix}$$

- Network Presentation

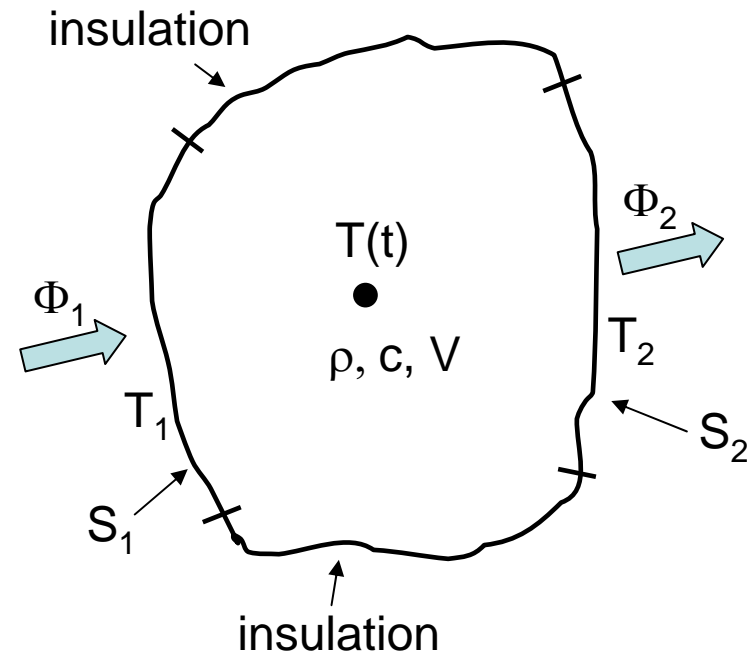


- Matrix representation



Case b: Transient-State

- Same system as before, but transient conditions
- At $t=0$, T uniform, Heat input $\Phi_1 \Rightarrow S_1$ heat output $\Phi_2 \Rightarrow S_2$ remaining S : insulated
- conductivity λ is large enough to assume, that $T(t)$ is uniform



- Heat balance:
$$\rho c V \frac{dT}{dt} = \Phi_1(t) - \Phi_2(t)$$

– Mass density: ρ , Specific heat: c , Volume: V , Heat capacity: $C_t = \rho c V$

Laplace transform of heat balance equation

- Laplace temperature: $\Rightarrow \Theta(p) = L[T(t)] = \int_0^{\infty} T(t) \exp(-pt) dt$

- Laplace heat flux: $\Rightarrow \Phi_i(p) = L[\Phi_i(t)] = \int_0^{\infty} \Phi_i(t) \exp(-pt) dt$

- Heat balance:

$$\rho c V \frac{dT}{dt} = \Phi_1(t) - \Phi_2(t)$$

$$\Rightarrow C_t p \Theta(p) = \Phi_1(p) - \Phi_2(p)$$

- Large conductivity λ

$$\Rightarrow T_1(t) = T_2(t) = T(t)$$

- Same is true for their Laplace transforms:

$$\Rightarrow \Theta_1(p) = \Theta_2(p) = \Theta(p)$$

Case b: Matrix and Network representation

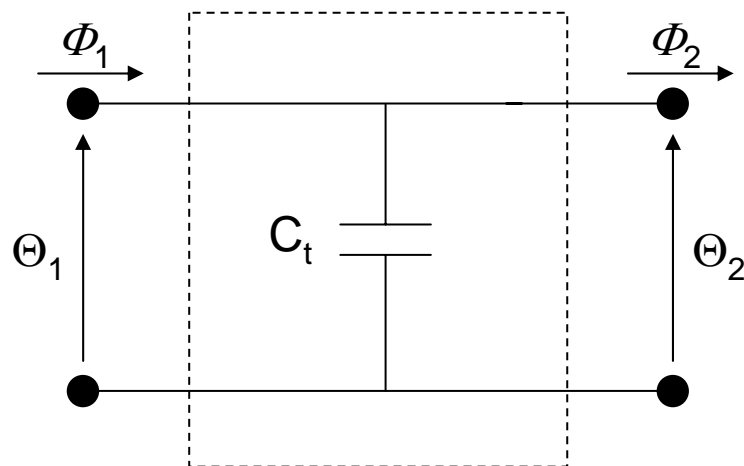
- Matrix equation form:

$$C_t p \Theta(p) = \Phi_1(p) - \Phi_2(p)$$

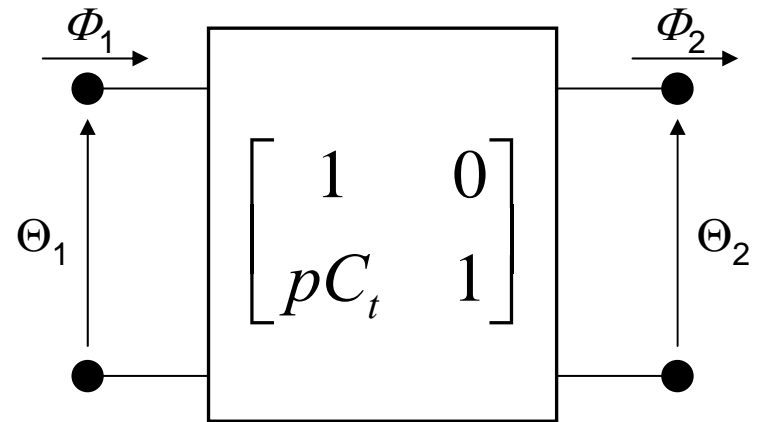
$$\Theta_1(p) = \Theta_2(p) = \Theta(p)$$

$$\begin{bmatrix} \Theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_t p & 1 \end{bmatrix} \begin{bmatrix} \Theta_2 \\ \Phi_2 \end{bmatrix}$$

- Network Presentation

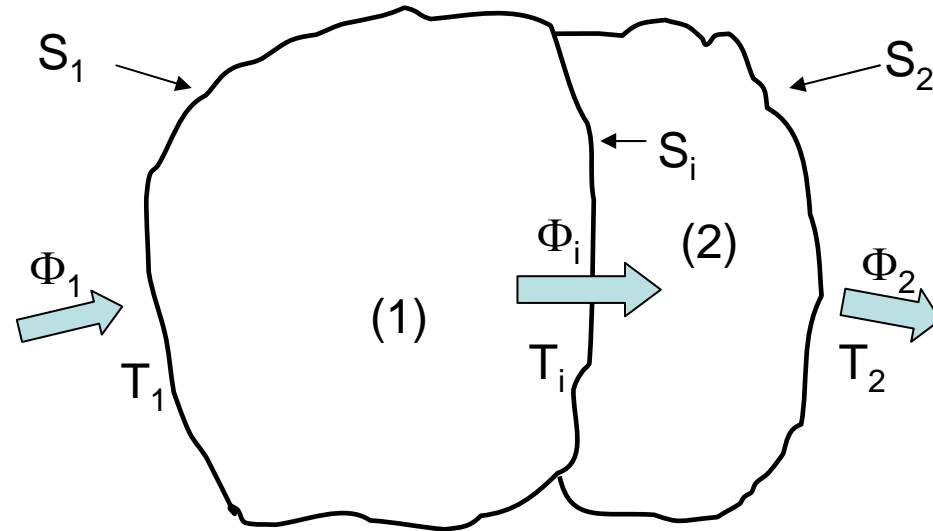


- Matrix representation



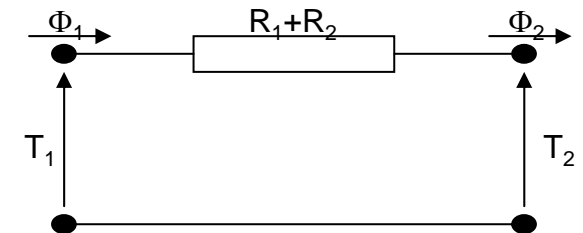
Case c: Cascading layers

- Cascade of two media



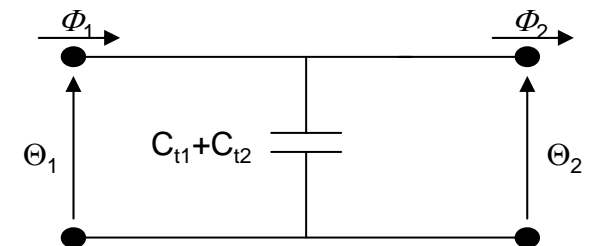
– Steady state:

$$\begin{bmatrix} T_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & R_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_2 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} 1 & R_1 + R_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_2 \\ \Phi_2 \end{bmatrix}$$



– Transient state:

$$\begin{bmatrix} \Theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_{t1}p & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ C_{t2}p & 1 \end{bmatrix} \begin{bmatrix} \Theta_2 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (C_{t1} + C_{t2})p & 1 \end{bmatrix} \begin{bmatrix} \Theta_2 \\ \Phi_2 \end{bmatrix}$$



Limitations

- Steady state:
 - average surface T (no local T),
 - external representation
- Transient state:
 - strong assumption of uniform temperature distribution
 - no geometrical data are used to calculate C_t (zero-dimensional model)
- Question: how it is possible to extend and combine steady state and transient state models?

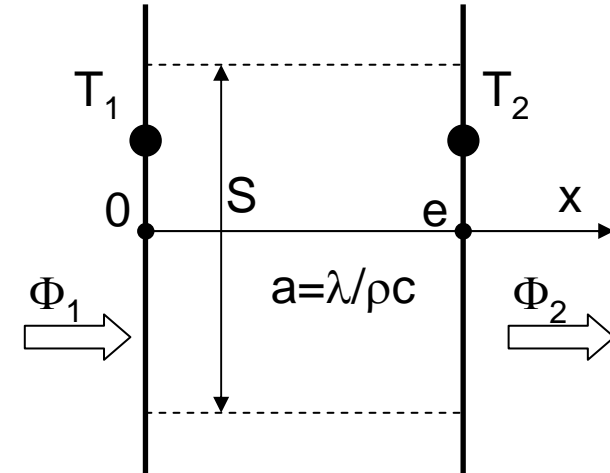
Case d: an infinite layer

- Things simplify a lot ...
 - Layer with thickness e ,
 - thermal conductivity λ ,
 - volumetric heat capacity ρc ,
 - mass density ρ , specific heat c ,
 - thermal diffusivity $a = \lambda / \rho c$,
 - cross section S
- 1-D Heat transfer: temperature field $T(x,t)$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \frac{\partial T}{\partial t} \quad \text{with} \quad T = T_0 \quad \text{for} \quad t = 0$$

- Heat flux Φ at any location x inside the layer:

$$\Phi = -\lambda S \frac{\partial T}{\partial x}$$



Laplace transforms of both temperature and flux

- Laplace temperature: $\Rightarrow \Theta(x, p) = \int_0^{\infty} T(x, t) \exp(-pt) dt$
- Laplace heat flux: $\Rightarrow \Phi_i(x, p) = \int_0^{\infty} \Phi_i(x, t) \exp(-pt) dt$
- Consider the Laplace transforms Θ_1 ($x=0$) and Θ_2 ($x=e$) and the transforms of the corresponding heat fluxes Φ_1 and Φ_2 : (it can be shown)

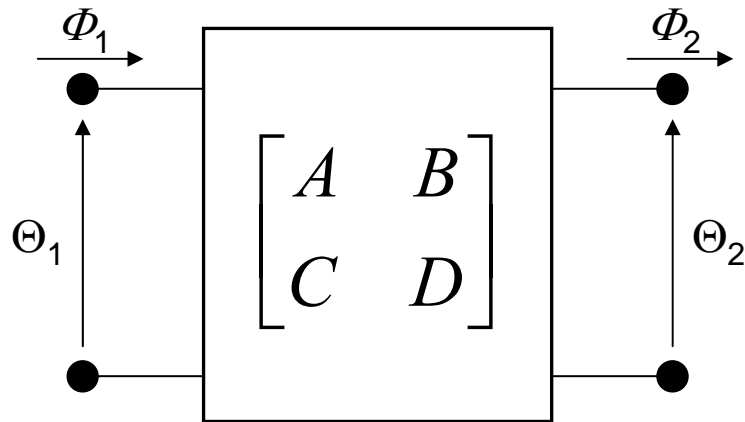
$$\begin{bmatrix} \Theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Theta_2 \\ \Phi_2 \end{bmatrix}$$

$$A = D = \cosh(ke) \quad B = \frac{1}{\lambda k S} \sinh(ke)$$

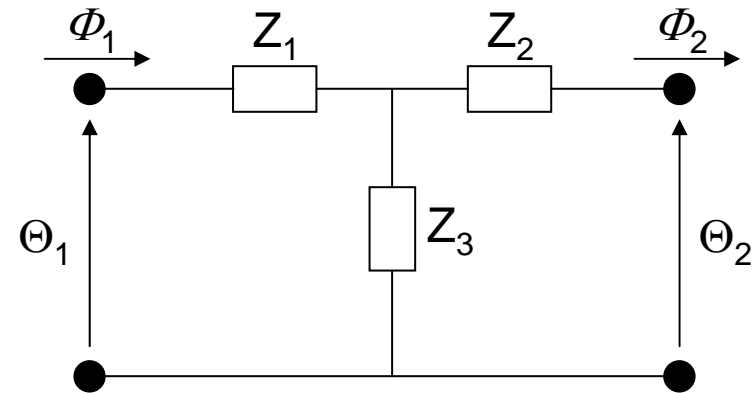
$$C = \lambda k S \sinh(ke) \quad k = \sqrt{p/a}$$

Case d: Matrix and Network representation

- The matrix completely characterizes the system
- It is valid for any boundary condition (in $x=0$ and $x=e$)
- It is strictly equivalent to the heat equation
- It relates to an external representation (Θ_1 , Θ_2 , Φ_1 and Φ_2 : are related to external surfaces)
- Matrix representation



Network representation



$$Z_1 = \frac{A-1}{C} \quad Z_2 = \frac{D-1}{C} \quad Z_3 = \frac{1}{C}$$

Case d: heat pulse response

- Assumption:
 - the front face is excited by a Dirac heat pulse of energy Q
 - The rear face remains insulated
- Translation into Laplace domain
- The system becomes:
- Calculation (plotting) in time domain
 - Inversion of the Laplace transform (e.g. numerical with Stehfest's algorithm)

$$\Phi = QS\delta(t) \quad \text{at } x = 0$$

$$\Phi = 0 \quad \text{at } x = e$$

$$\Phi = QS = \Phi_1 \quad \text{at } x = 0$$

$$\Phi = 0 = \Phi_2 \quad \text{at } x = e$$

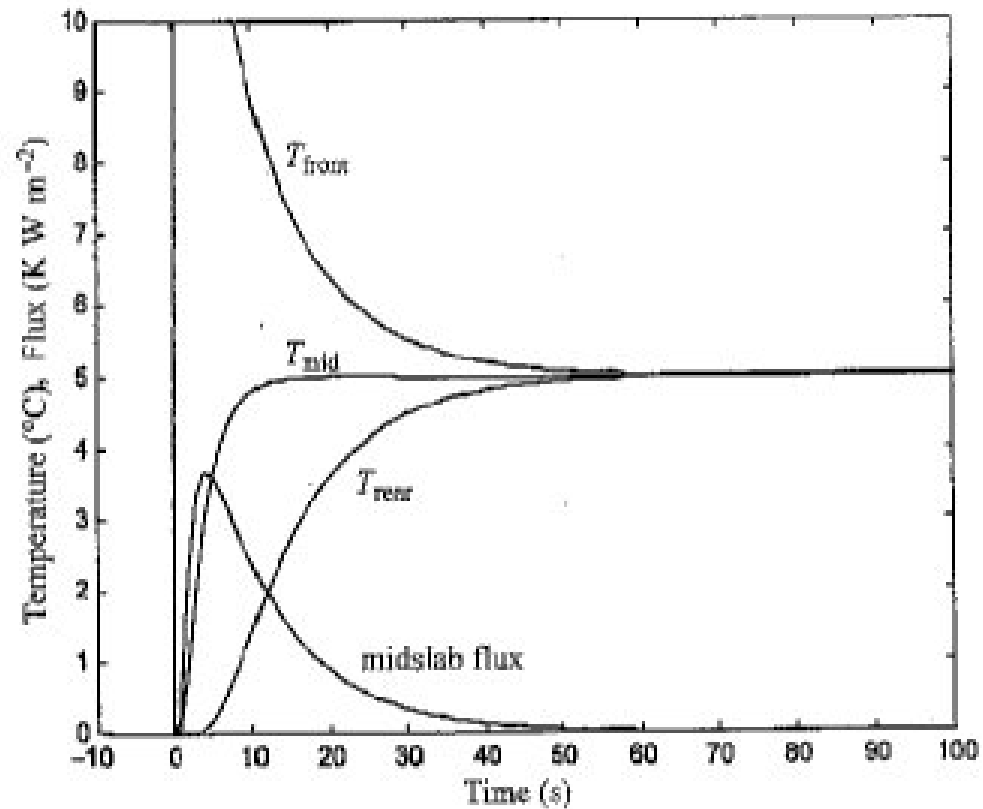
$$\Theta_1 = A\Theta_2 + B\Phi_2 \quad \Phi_1 = C\Theta_2 + D\Phi_2$$

$$\Theta_2 = \frac{QS}{C} = \frac{Q}{\lambda\sqrt{p/a} \sinh(e\sqrt{p/a})}$$

$$\Theta_1 = \frac{QSA}{C} = \frac{Q}{\lambda\sqrt{p/a} \tanh(e\sqrt{p/a})}$$

Time domain

- Temperature response of a one-layer excited by a Dirac heat flux



Impedance

Small and large times asymptotes

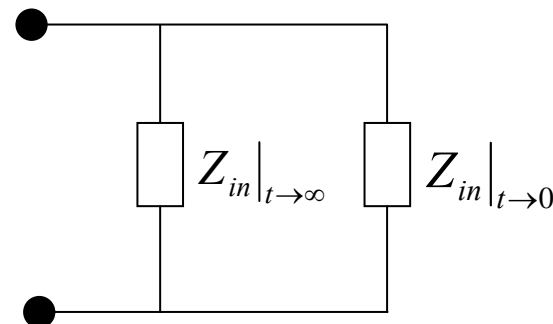
- Analytical expression:
Impedance seen from the input under the $\Theta_2=0$ boundary condition at the output (thermochuck)
- Short times solutions:
 - Transmission line model
 - Recursive network
- Long times solutions:
 - R_{th}
- Network presentation

$$Z_{in} = \frac{\Theta_1}{\Phi_1} \Big|_{\Theta_2=0} = \frac{B}{D} = \frac{\tanh(e\sqrt{p/a})}{\lambda S \sqrt{p/a}}$$

$$\text{or } Z_{in} = Z_1 + Z_2 // Z_3$$

$$t \rightarrow 0 \quad Z_{in} \Big|_{p \rightarrow \infty} = \frac{1}{S \sqrt{\lambda \rho c} \sqrt{p}}$$

$$t \rightarrow \infty \quad Z_{in} \Big|_{p \rightarrow 0} = \frac{e}{\lambda S} = R_{th}$$

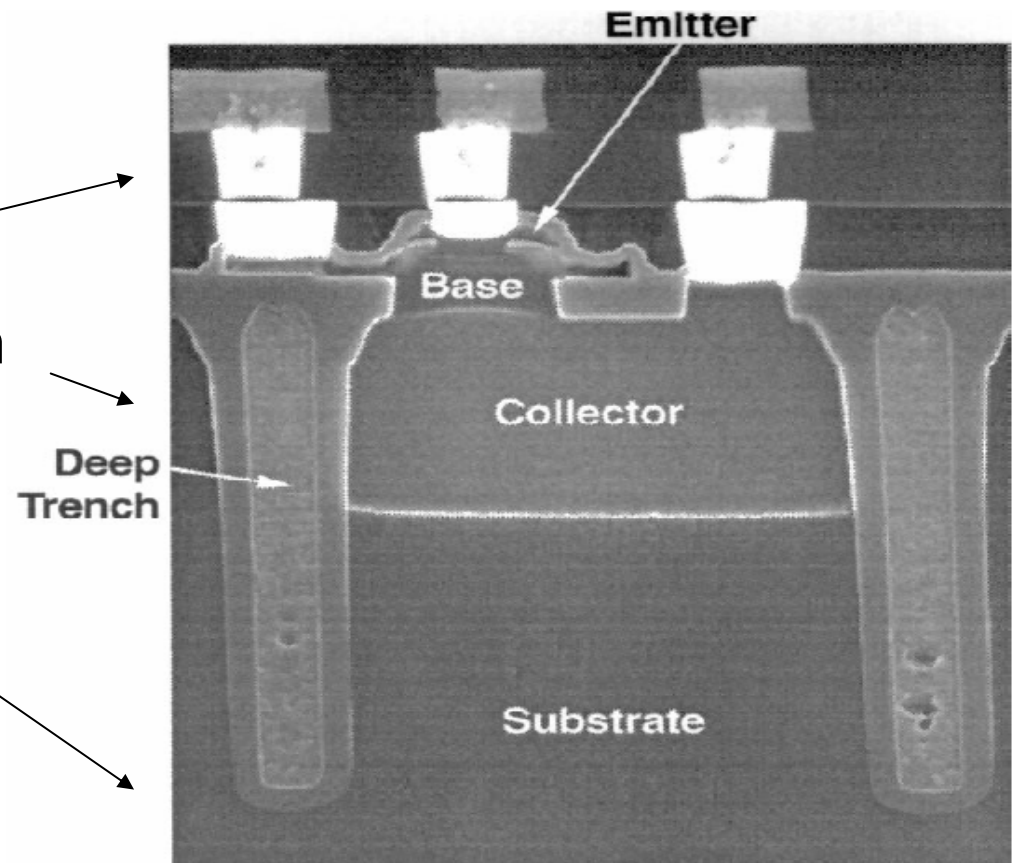


Application to Si/SiGe HBT

- Cross section:

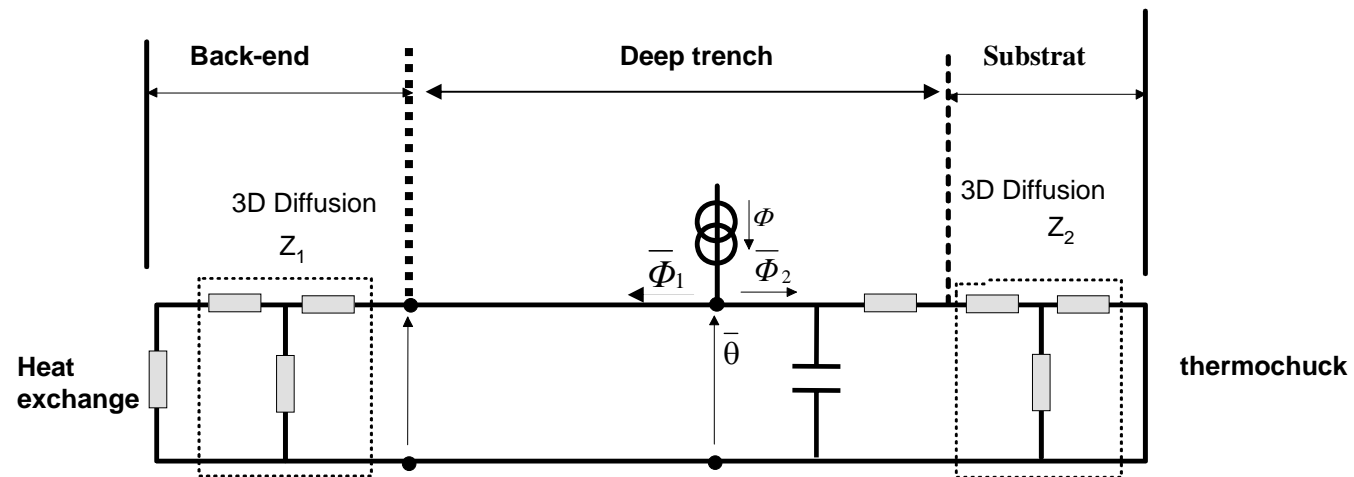
- 3 layers

- Back end
 - Insight deep trench with heat source
 - substrate



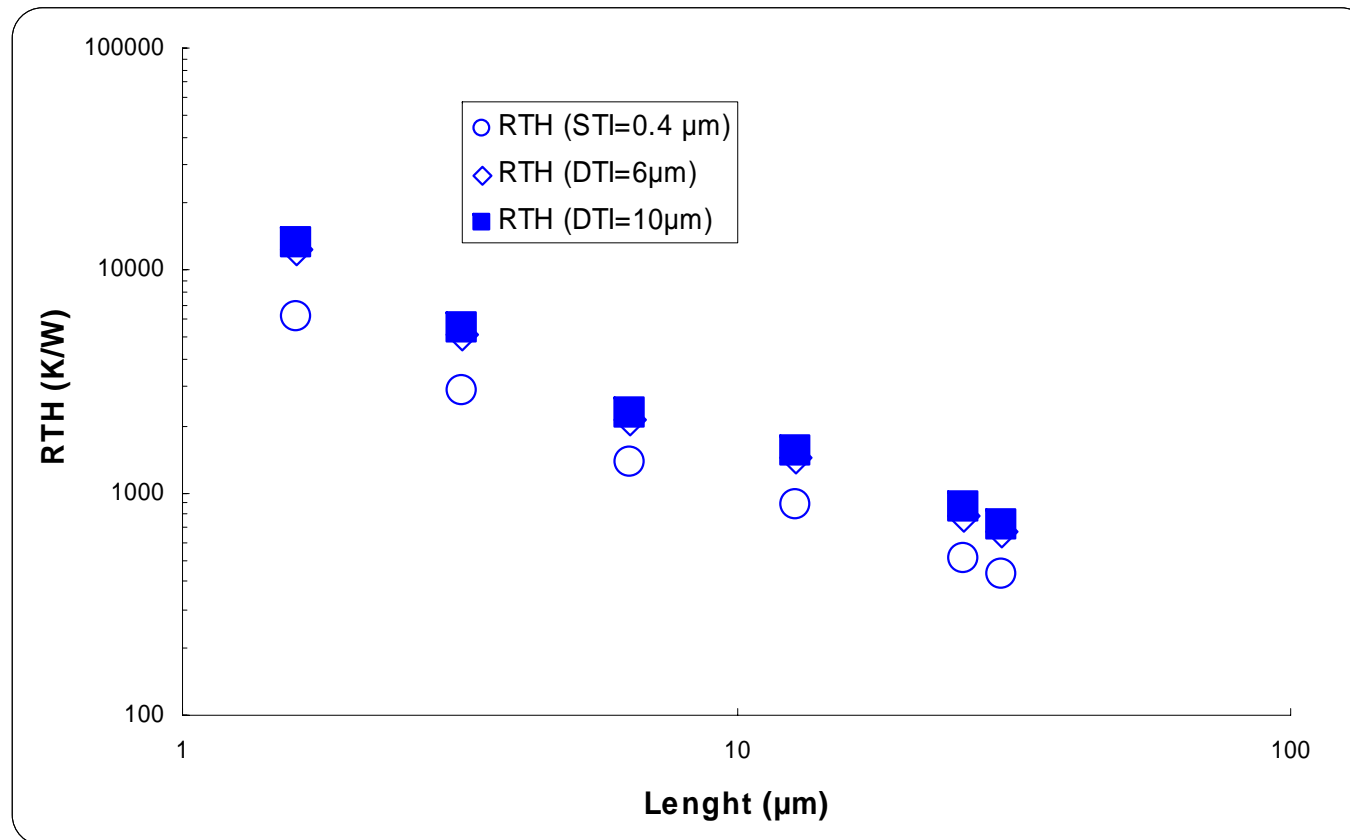
Quadrupole representation for HBT

- Thermal schematic



Quadrupole analysis: results (1)

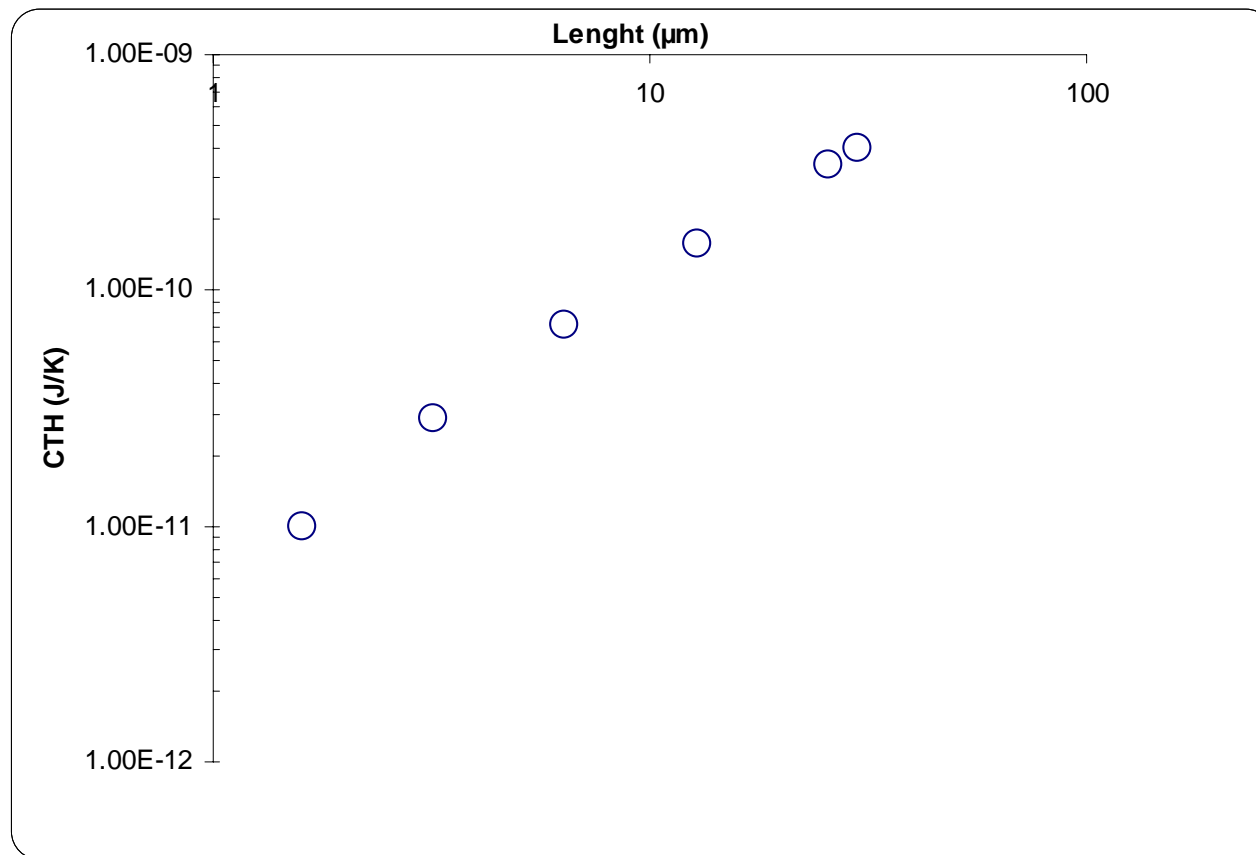
- Steady state: length and DTI scaling



Drawn Emitter width : 0,4 μm

Quadrupole analysis: results (2)

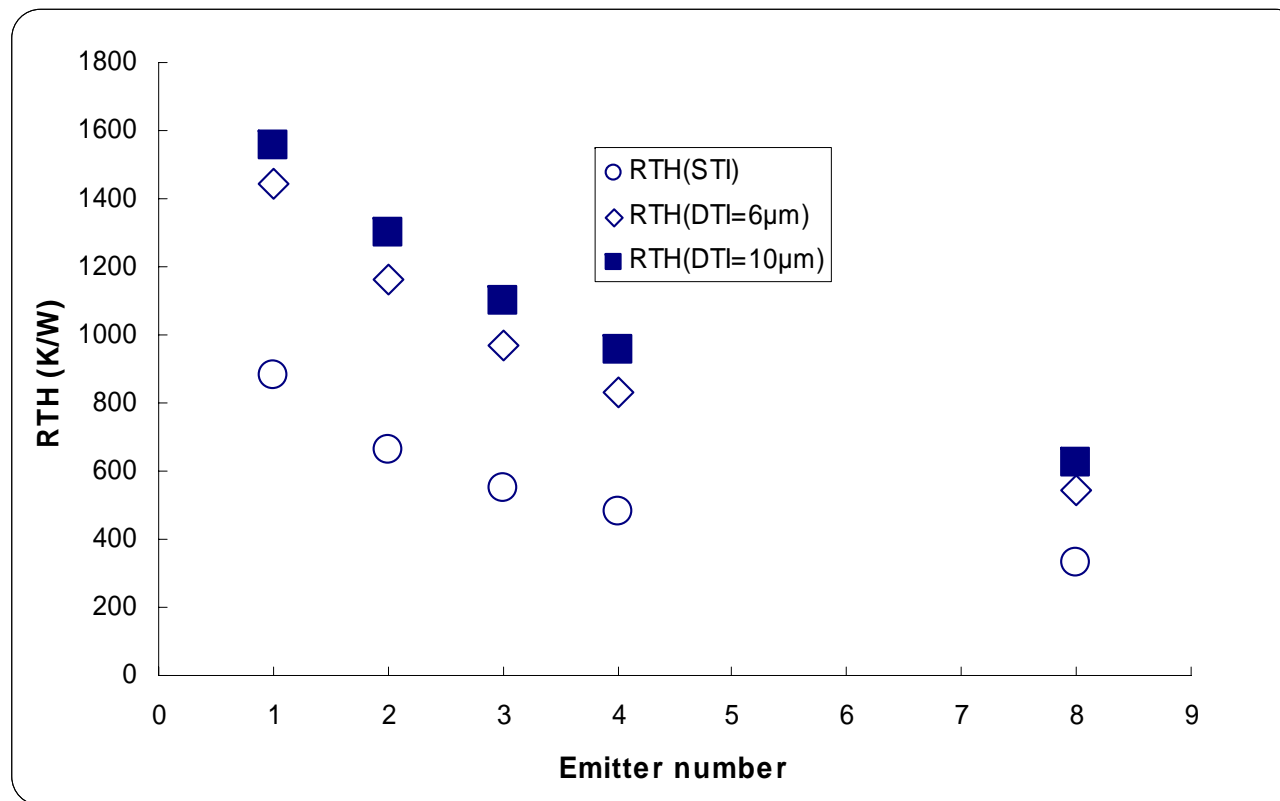
- Transient state: length scaling



Drawn Emitter width : 0,4μm

Quadrupole analysis: results (3)

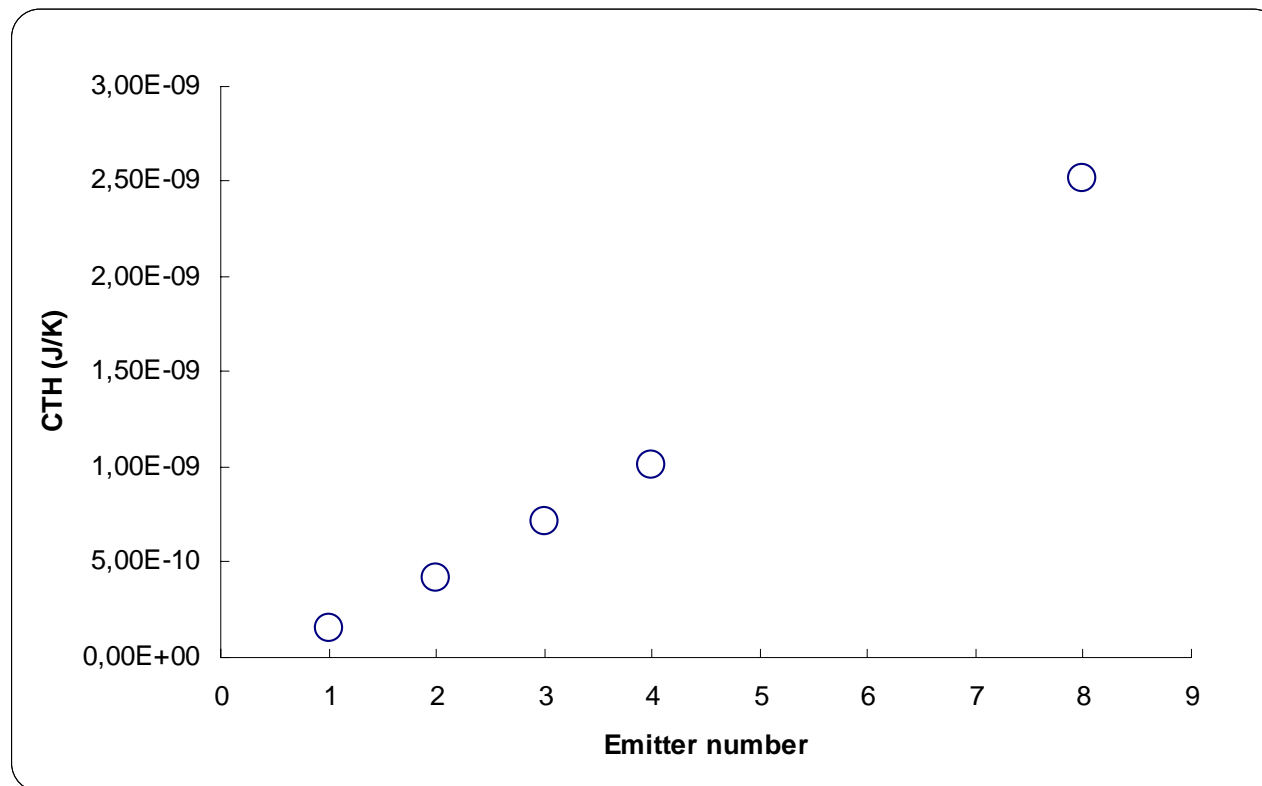
- Steady state: Emitter number and DTI scaling



Drawn Emitter Surface : $30 \times 0,4 \mu\text{m}^2$

Quadrupole analysis: results (4)

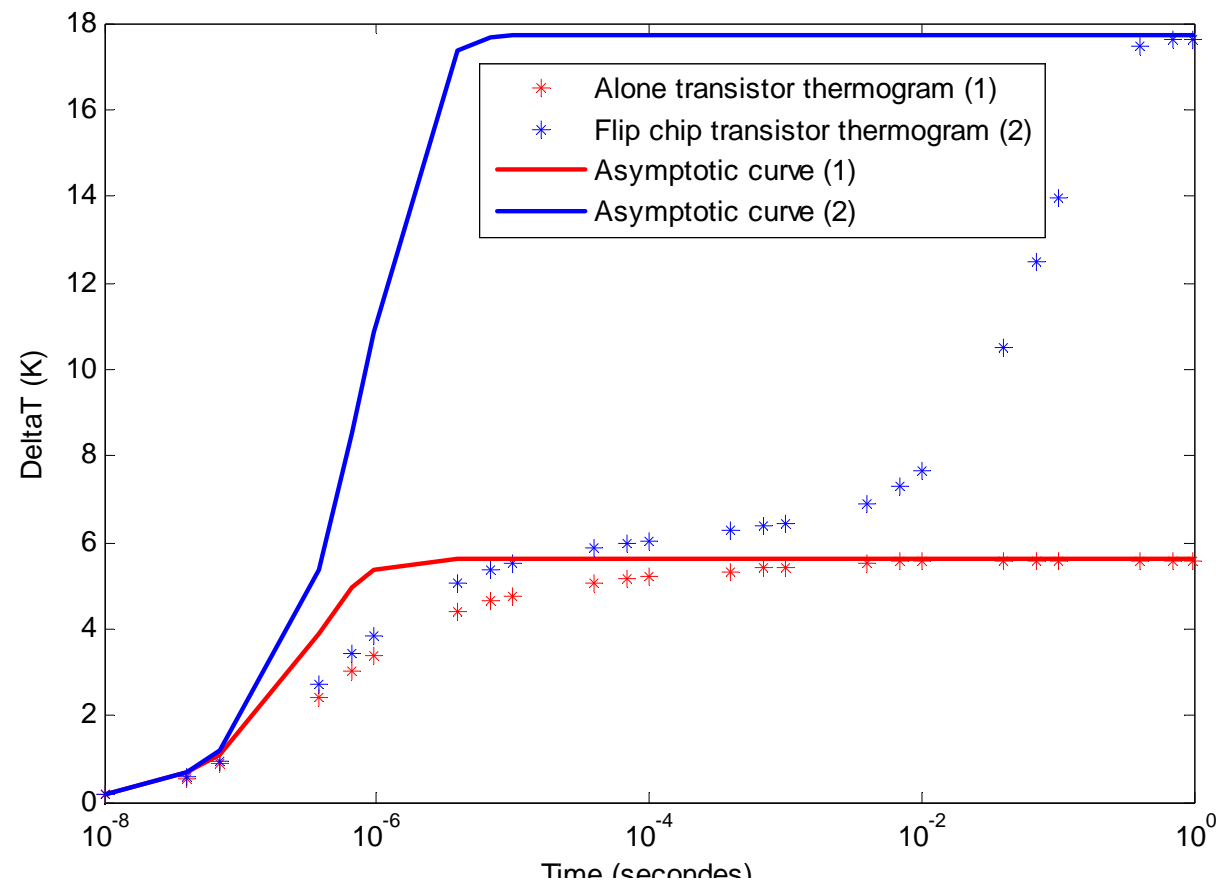
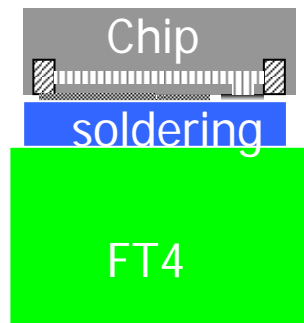
- Transient state: Emitter number scaling



Drawn Emitter Surface : $30 \times 0,4 \mu\text{m}^2$

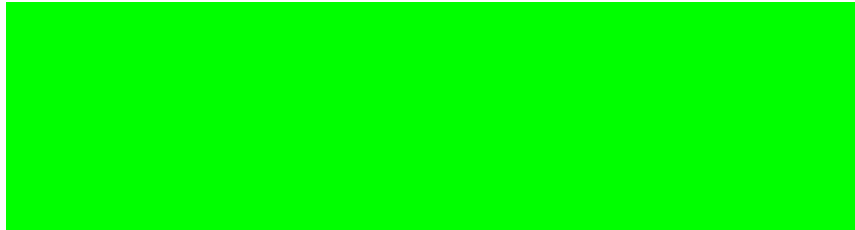
Flip chip: result

- Proof of concept



Toolkit

In one click...



Component description

LAYOUT

EMITTER FINGER

L1 (length, μm)	W1 (width, μm)	Spacer (Width, μm)	a1 (μm)	a2 (μm)
30	0.4	0.075	0.4	0.7
			a3 (μm)	a4 (μm)
			2	0.7

Single emitter device
 Multiple emitter device

	1	E
	1	B
	1	C

TECHNOLOGY

BACK END: METALLISATION

Vias Characteristics

E	Length, μm	0.2	Vwidth, μm	0.2	Number	16
B	Length, μm	0.2	Vwidth, μm	0.2	Number	16
C	Length, μm	0.2	Vwidth, μm	0.2	Number	16

Thermal Cond.	Heat Capacity	Thermal diffusivity	Depth (μm)
Other...	66	2.3409e+006	2.81943e-005
9.55			

OTHER TYPE OF METALLISATION

DEEP TRENCH

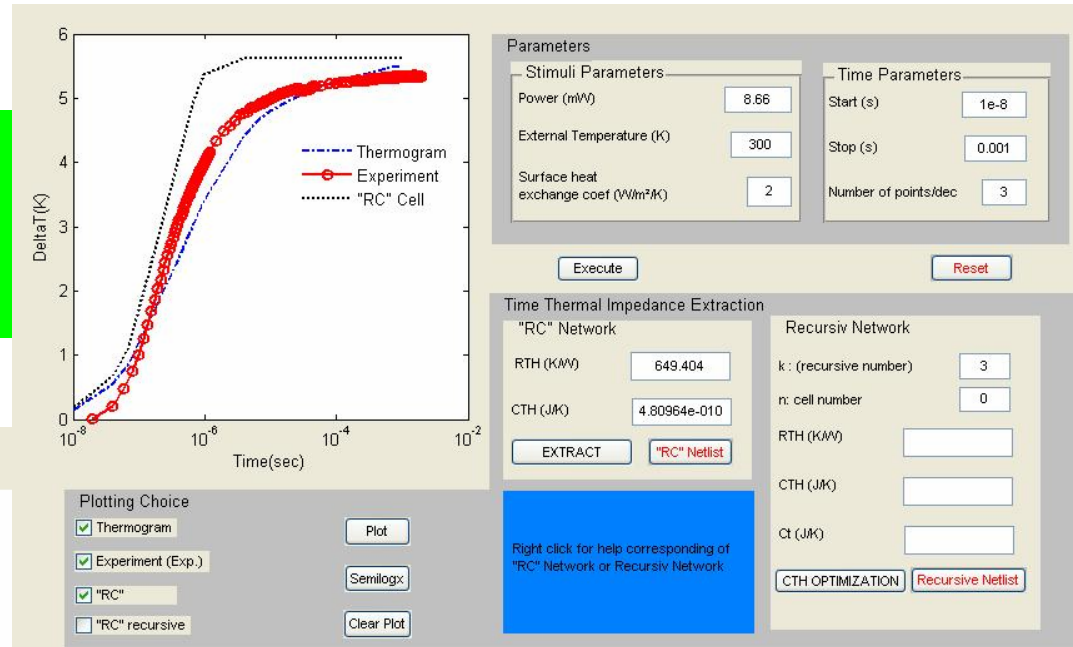
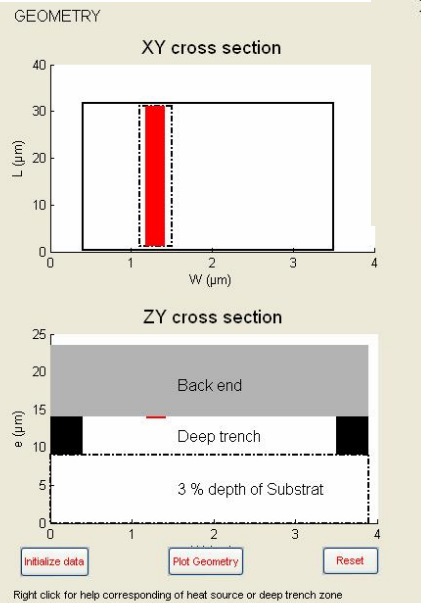
Thermal Cond.	Heat Capacity	Thermal diffusivity	Depth (μm)
Silicon	148	1.631e+006	9.07419e-005
5			

SUBSTRATE

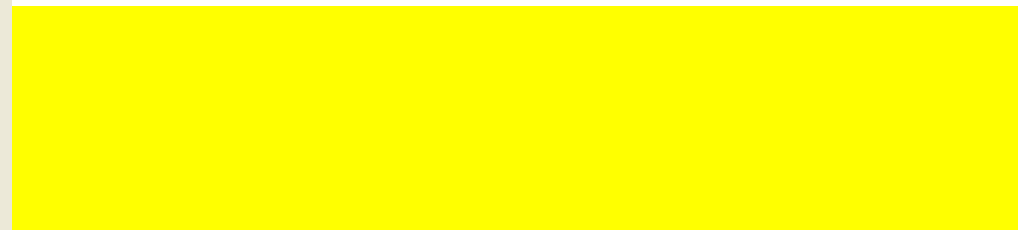
Thermal Cond.	Heat Capacity	Thermal diffusivity	Depth (μm)
Si	148	1.631e+006	9.07419e-005
300			

COUPLING CELL

No Yes



Results



Discussion

- Quadrupole network for heat exchange modeling
- Analytical results: fast and accurate
- External approach (no internal temperature field)
- Thermal impedance
- Well adapted for compact modeling
- Extensible for specific configurations
 - Multi-emitter fingers
 - Multi-cells (PA)
 - Mutual coupling (under work)
 - Packaging
 - Flip chip
- Versatile toolkit

One minute left

- References

- Thermal quadrupoles, solving the heat equation through integrals transforms, D. Mailet, S. André, J.C. Batsale A. Degiovanni, C. Moyne, Ed. Wiley, 2000
- Pierre-Yvan Sulima, *Contribution à la modélisation thermique des transistors bipolaires à hétérojonction SiGe*, thesis, Université Bordeaux 1, defense : 13 décembre 2005
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- Helene Beckrich, *Caractérisation, modélisation et conception de transistors RF de puissance intégrés dans une filière BiCMOS submicronique*, thesis, Université Bordeaux 1, defense : 27 november 2006
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- Acknowledgement

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- Nano2008, Minefi, Ministère de l'économie, des finances et de l'emploi (French ministry of economy, finance and work)

END

- Thank you for your attention