Estimation of fmax by the Common Intercept Method

Z. Huszka, K. Molnár, E. Seebacher

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Purpose

Reduce Uncertainty of *fmax* Estimation

from Limited Upper Frequency Measurements

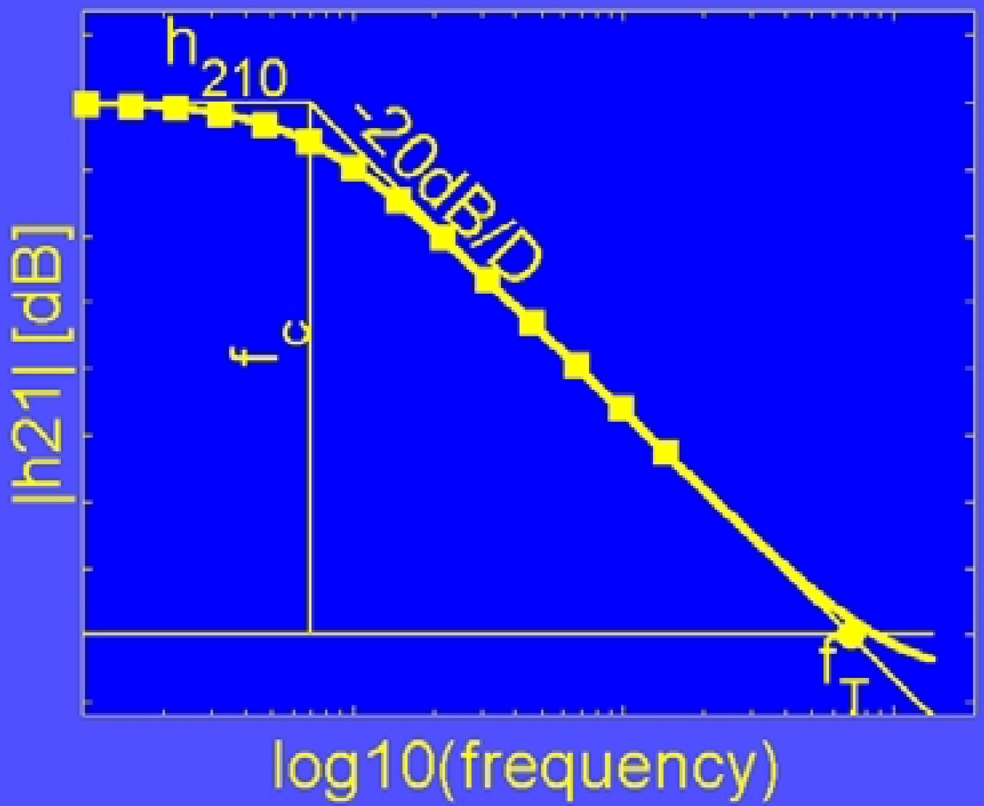
by Introducing

- New Functions with the Same *fmax* Intercept
- Constrained Extrapolation

Outline

- Overview of f_T and *fmax* Estimations
- Passivity Functions
- Approximation to Passivity Functions
- Unconstrained Extrapolation
- Constrained Extrapolation
- Extrapolation Error
- Conclusions

f_T



■ measurement points

-20dB/D technique:

large measurement frequencies are needed

Definition:

$f_T = f_C * h_{210}$

Extract h_{210} and f_C!

(f_C<3GHz)

Power Gain

$$UPG = \frac{1}{4}\frac{\left|z_{21} - z_{12}\right|^2}{\Re(z_{11})\Re(z_{22}) - \Re(z_{12})\Re(z_{21})}$$

$$MAG = MSG \cdot \left(k - \sqrt{k^2 - 1}\right) \qquad MSG = \left|\frac{\gamma_{21}}{\gamma_{11}}\right|$$

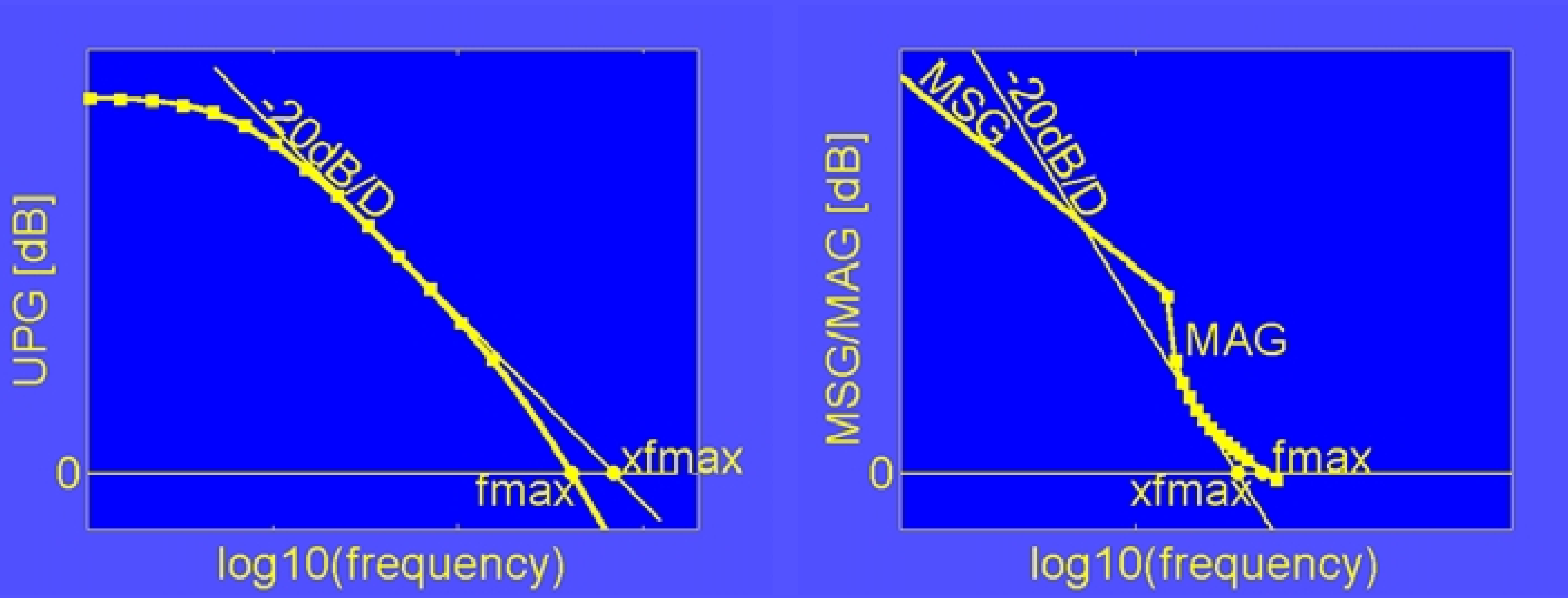
$$k = \frac{2\Re(\gamma_{11})\Re(\gamma_{22}) - \Re(\gamma_{12}\gamma_{21})}{\left|\gamma_{12}\gamma_{21}\right|}$$

UPG and **MAG** provide the same unity gain intercept

f_{max} by -20dB/D

Overestimation using UPG

Underestimation using MAG



- ■ measurement points
- ● unity gain intercepts

Passivity

Transition point between passive and active modes:

det(Γ+Γ^H)=0

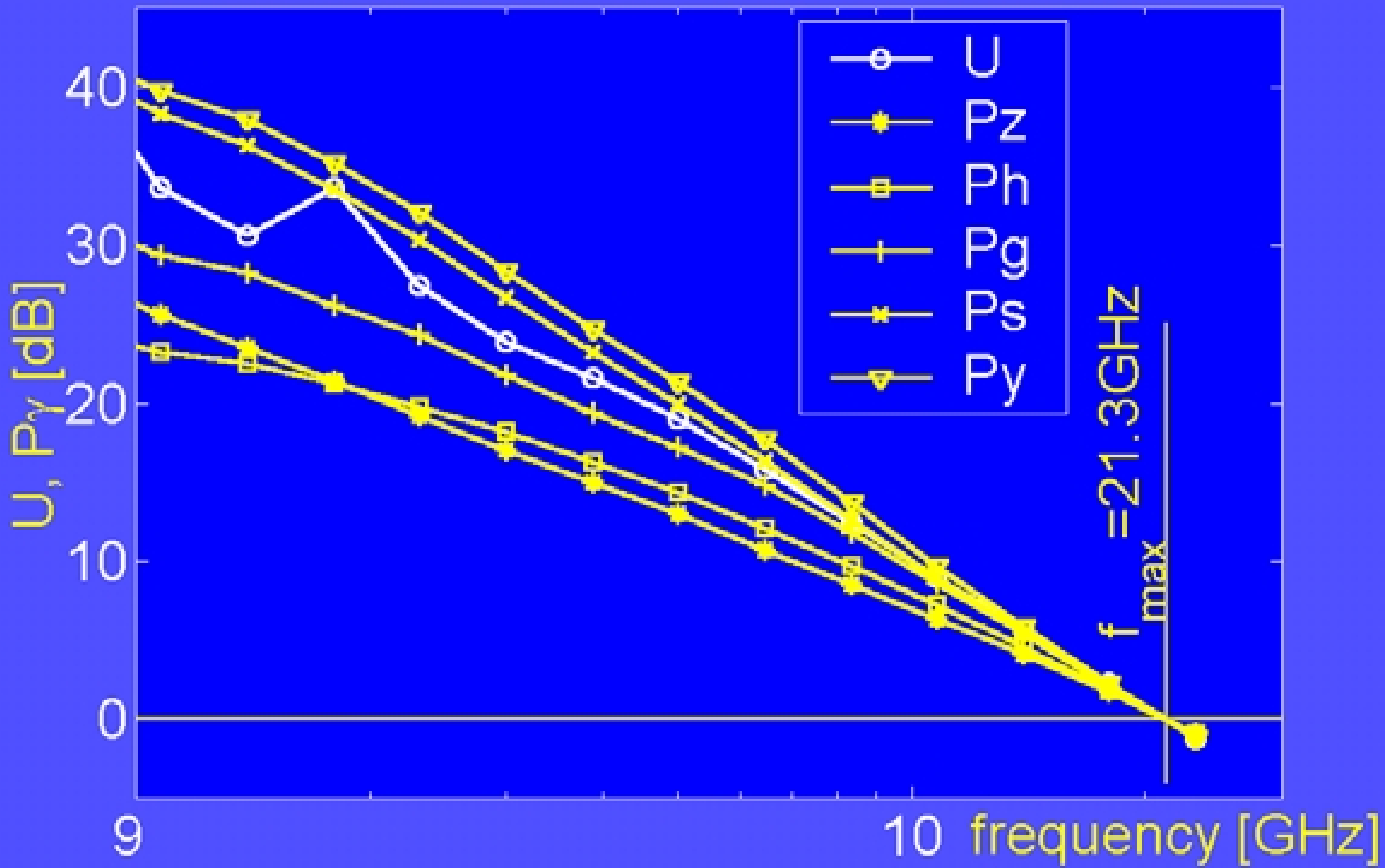
Two-port parameter matrix Γ can be *Z, Y, H* or *G.*

PASSIVITY FUNCTION

$$P_\gamma = \frac{\left|\gamma_{21} + \gamma_{12}^*\right|^2}{4\Re(\gamma_{11})\Re(\gamma_{22})}$$

P_s for *S* parameters is different

Common Intercept



6 functions meet in *fmax*

Concept

- Approximation to passivity functions
- Selection of basis functions
- Smoothing functions with tangential extension
- Unconstrained extrapolation to *fmax*
- Constrained extrapolation to *fmax* **(preferred)**
- Error estimation

Approximation

P_γ are rational functions in ω^2

$$\frac{P_\gamma}{\Omega} = \frac{\alpha_{n-1}\Omega^{n-1} + \alpha_{n-2}\Omega^{n-2} + \cdots + \alpha_0}{\beta_n\Omega^n + \beta_{n-1}\Omega^{n-1} + \cdots + \beta_0} \qquad \Omega = \left(\frac{f_M}{f}\right)^2$$

f_M=max. measurement frequency

Polynomial approximation y(x) with transformed variables

$$x = \log(\Omega) \qquad y = \log\left(\frac{P_\gamma}{\Omega}\right)$$

Basis Functions

Gram polynomials $p_k(x)$ of order k and l satisfy

$$\sum_{i=1}^{N} w_i p_k(x_i) p_l(x_i) = (w_i, p_{ki}, p_{li})_i = \delta_{kl} \quad (\text{Kronecker's } \delta)$$

with weights w_i in measurement points x_i ($i=1, 2, \ldots N$)

Smoothing function with coefficients a_k

$$\tilde{y}_i = (a_k, p_{ki})_k$$

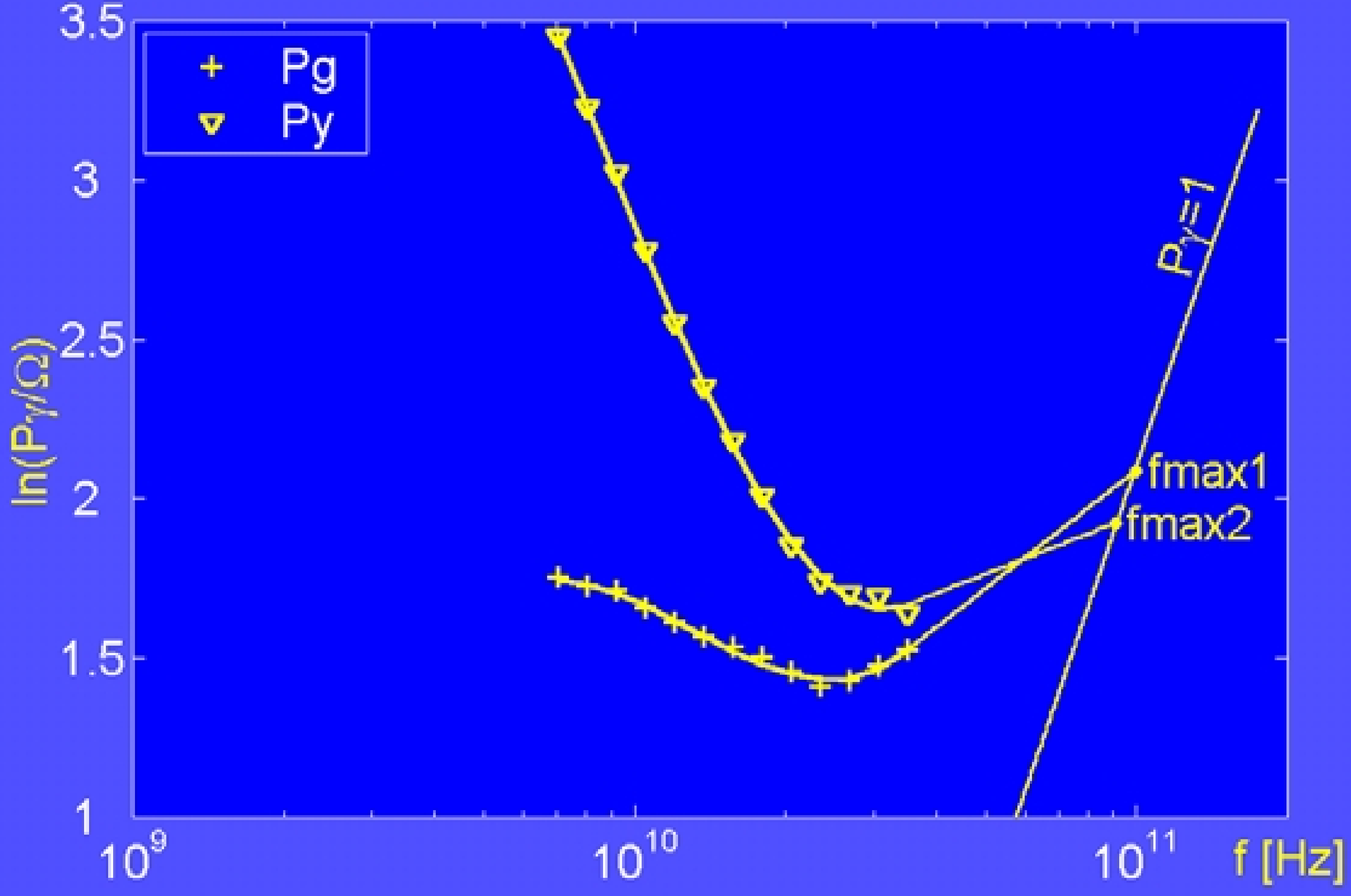
Unconstrained Extrapolation

Find best fit over measurements

$$S_\gamma = \frac{1}{2}\left(w_i(\tilde{y}_i - y_i)^2\right)_i = \min$$

- extend smoothing function by its tangent in last measurement point to unity gain
- calculate fmax as the average of the unity gain intercepts belonging to different passivity functions

Unconstrained Extrapolation *(cont.)*



fmax ≈ mean(fmax1, fmax2)

Constrained Extrapolation

Find best fit over measurements with constraint

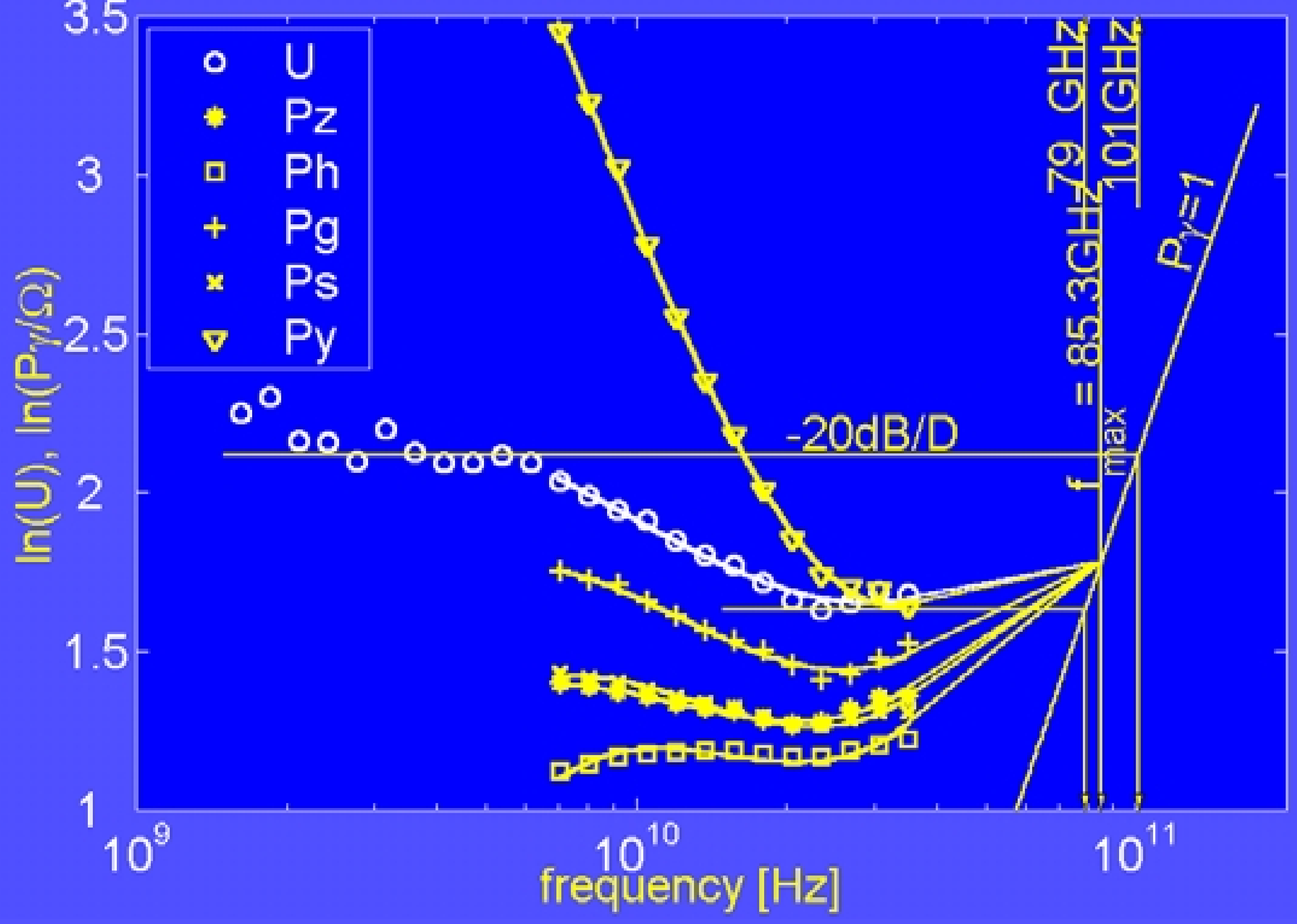
$$S_\gamma(x_m) = \frac{1}{2}\left(w_i(\tilde{y}_i - y_i)^2\right)_i + \lambda c_1(x_m) \qquad x_m = \log\left(\frac{f_M}{f_{\max}}\right)^2$$

- λ Lagrange multiplier
- $c_1(x_m)$ forces linear extension cross the 0dB line in x_m

Minimize the sum of errors in x_m

$$S(x_m) = S_U(x_m) + S_Z(x_m) + S_Y(x_m) + S_H(x_m) + S_G(x_m) + S_S(x_m)$$

Constrained Extrapolation *(cont.)*



Physical property of common intercept is fully utilized

Error Estimation

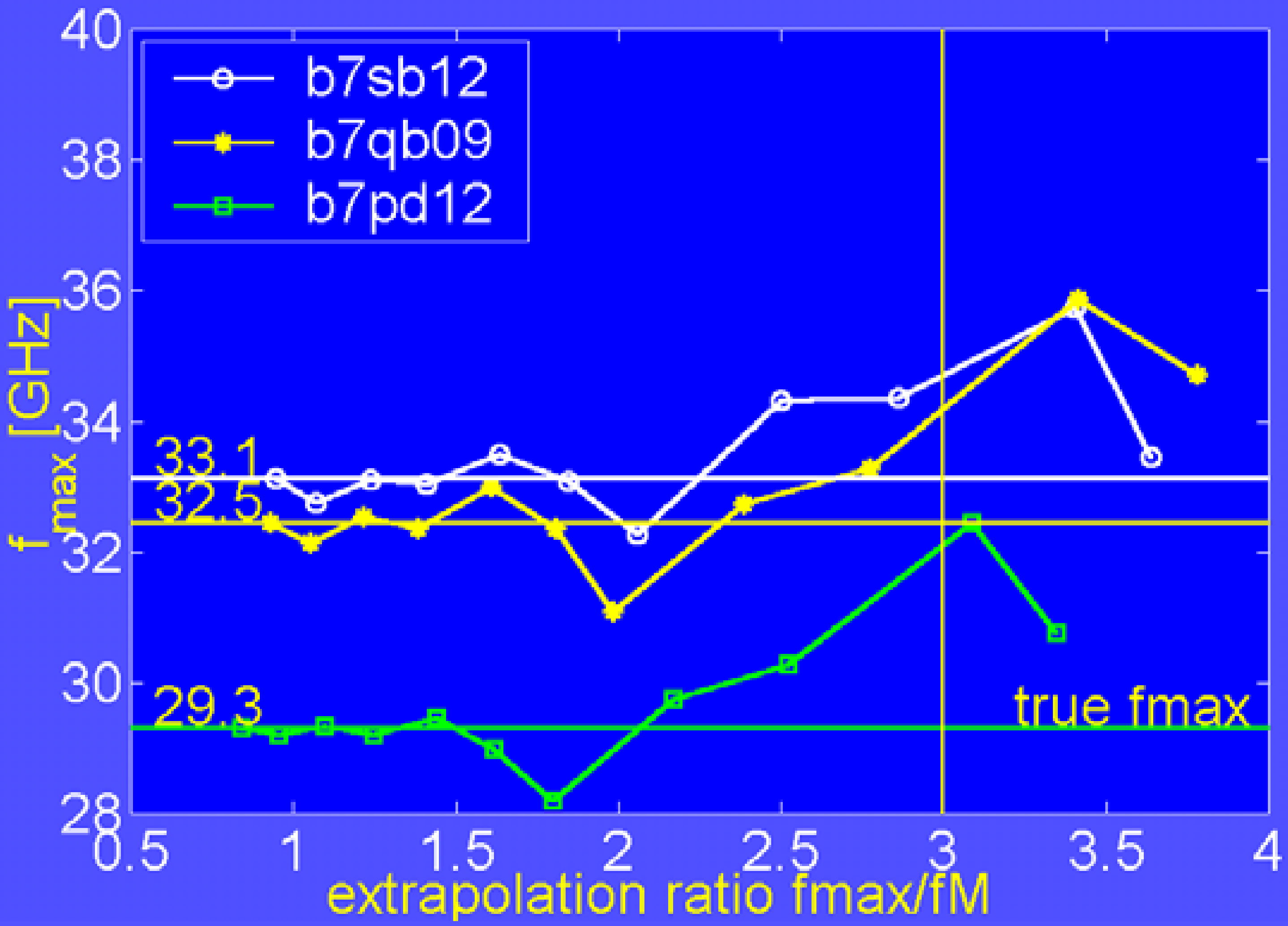
Method 1

- Select device with fmax below the measurement limit of the VNA ($f_{max}<f_M$) (fmax with interpolation)
- Glide down to $f_M<f_{max}$ and observe extrapolation error

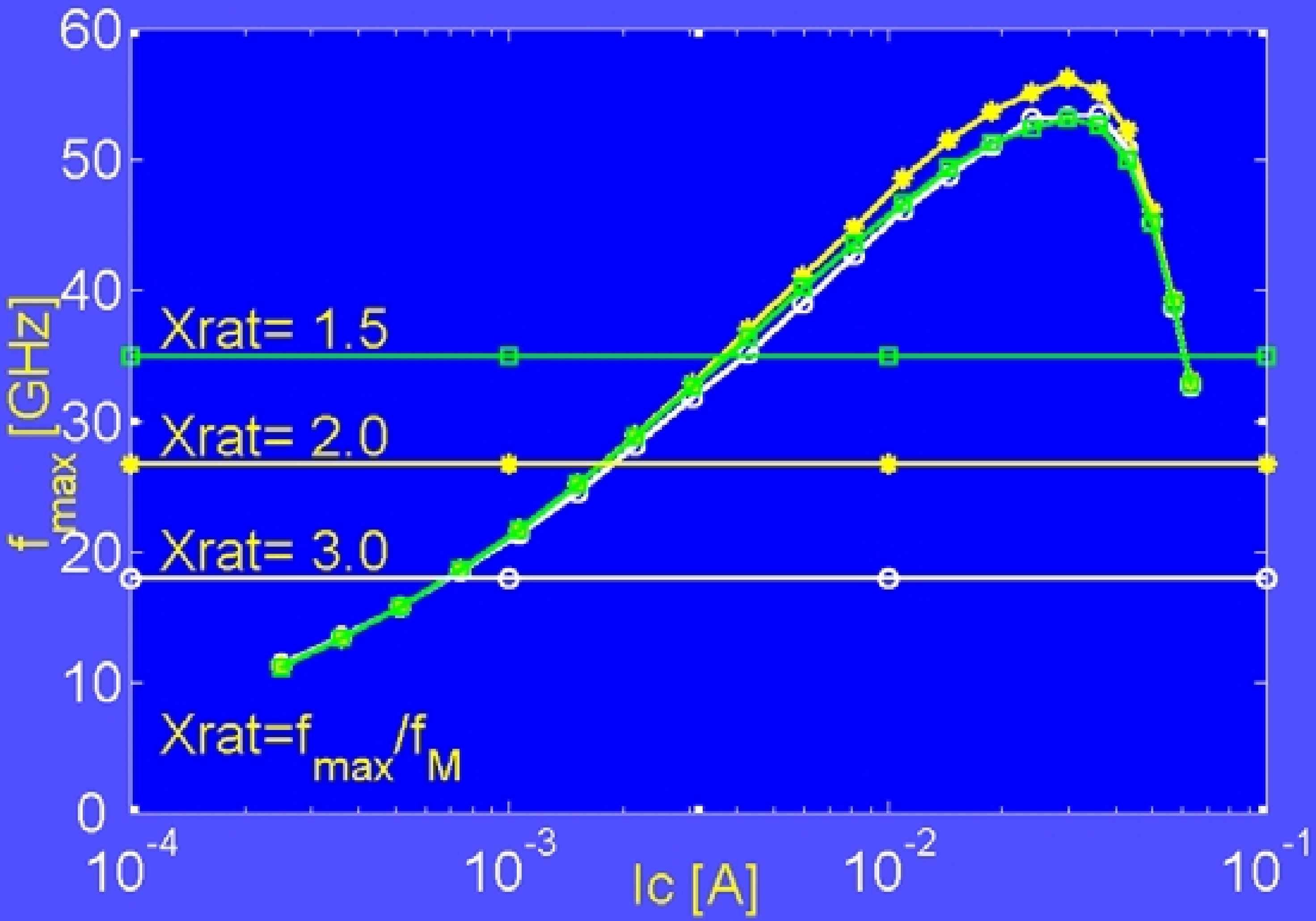
Method 2

- Select device with $f_{max}>f_M$
- Compare fmax curves at different f_M selections

Extrapolation Error *(Method 1)*



Extrapolation Error *(Method 2)*



Conclusions

- U and MAG do not follow a -20dB/D slope near unity gain leading to errors in fmax estimation.
- New passivity functions have been introduced with the same unity gain intercept as U and MAG.
- A constrained extrapolation method has been suggested to increase the robustness of fmax determination.
- Estimations are reasonable up to $f_{max}/f_M<3$ and excellent below $f_{max}/f_M=2.5$