Method for the Determination of the Optimal Noise Parameters from SPICE Simulations

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Purpose

• Method for extending circuit simulators’ capability to perform direct noise parameter determination
• Integrating the calculus in CADENCE environment
Outline

• Noise analysis limitation in recent circuit simulators
• Noise correlation basics
• Determination of the missing cross correlation term
Limitations

- general noisy two-port has correlated noise sources
- circuit simulators compute an autocorrelation term only
- noise parameters can not be directly obtained in absence of cross-correlation information
- suggested optimization is long and inconvenient to configure
- designers’ choice: simplified models for input transistor
Noise Sources in VBIC
State-of-Art Noise Evaluation

- Nodal admittance matrices of adjoints are transposes (adjoint: controlled source configurations “inverted”)
- Transfer impedance calculated by interreciprocity
- Transferred noise squares are superimposed
Evens and Odds

Efficiencies

• Inverse matrix of transpose is the transpose of the inverse: only one matrix inversion is necessary
• Determination of all transfer impedance is performed in one step
• Noise calculation is reduced to a set of multiplication followed by superposition at the output

Deficiencies

• Correlation of noise sources (e.g. I_b-I_c) cannot be handled
• *Optimization* is to be used for noise parameter determination
Two-Port Noise

Noise is propagating to terminals through internal network elements

Internal noise represented by two *correlated* terminal noise sources

Small signal parameter (SSP) description is necessary but insufficient
Noise Correlation Matrix (NCM)

\[
C_Z = \frac{1}{2\Delta f} \left< \begin{bmatrix} v_{Z1} \\ v_{Z2} \end{bmatrix} \cdot \begin{bmatrix} v_{Z1}^* \\ v_{Z2}^* \end{bmatrix}^H \right> = \frac{1}{2\Delta f} \left< \begin{bmatrix} v_{Z1} \\ v_{Z2} \end{bmatrix} \cdot \begin{bmatrix} v_{Z1}^* & v_{Z2}^* \end{bmatrix} \right> = \frac{1}{2\Delta f} \begin{bmatrix} \left< v_{Z1} v_{Z1}^* \right> & \left< v_{Z1} v_{Z2}^* \right> \\ \left< v_{Z2} v_{Z1}^* \right> & \left< v_{Z2} v_{Z2}^* \right> \end{bmatrix}
\]

- Diagonal elements: autocorrelation of equivalent noise sources (real powers)
  \[\left< v_{Z1} v_{Z1}^* \right>, \left< v_{Z2} v_{Z2}^* \right>\]

- Off-diagonals: cross-correlation of equivalent noise sources (complex powers)
  \[\left< v_{Z1} v_{Z2}^* \right>, \left< v_{Z2} v_{Z1}^* \right>\]

- Noiseless two-ports: one single SSP matrix
- Noisy two-ports: a pair of NCM and SSP matrices

Elements of \(C_Z\): power spectral densities (PSD, \(S_x\))
\([V^2/Hz], [A^2/Hz] \) or \([VA/Hz]\)
set \( i_1 = i_2 = 0 \) in \( Z \) and substitute the voltage vector in \( Y \)

\[
\begin{bmatrix}
\nu_1 \\
\nu_2 
\end{bmatrix} = Z \begin{bmatrix}
i_1 \\
i_2 
\end{bmatrix} + \begin{bmatrix}
\nu_{Z1} \\
\nu_{Z2} 
\end{bmatrix}
\]

\[
\begin{bmatrix}
i_1 \\
i_2 
\end{bmatrix} = Y \begin{bmatrix}
\nu_1 \\
\nu_2 
\end{bmatrix} + \begin{bmatrix}
\nu_{Y1} \\
\nu_{Y2} 
\end{bmatrix}
\]

General transformation \( P \rightarrow Q \)

\[
C_Q = T_{P2Q} C_P T_{P2Q}^H
\]
Concept

compute the two autocorrelation terms of Parent

compute one autocorrelation term of Child#1

compute one autocorrelation term of Child#2

connect values by NCM transformation formulae
Details

**NCM transformation O to P**

\[
C_P = T_{O2P} C_O T_{O2P}^H
\]

\[
T_{O2P} = \begin{bmatrix}
    t_{p11} & t_{p12} \\
    t_{p21} & t_{p22}
\end{bmatrix}
\]

Parent: \[
2\Delta fC_O = \begin{bmatrix}
    d_{o11} & d_{o12} \\
    d_{o21} & d_{o22}
\end{bmatrix}
\]

SPICE returns \( d_{o11}, d_{o22} \)

\( d_{p11} \) can be obtained from Child#1 by SPICE.

Alternatively, by NCM transformation from Parent:

\[
d_{p11} = d_{o11} |t_{p11}|^2 + 2 \Re(d_{o12} t_{p11} t_{p12}^*) + d_{o22} |t_{p12}|^2
\]

\( d_{q22} \) can be obtained from Child#2 by SPICE.

Alternatively, by NCM transformation from Parent:

\[
d_{q22} = d_{o11} |t_{q21}|^2 + 2 \Re(d_{o12} t_{q21} t_{q22}^*) + d_{o22} |t_{q22}|^2
\]
Unknown off-diagonal element $d_{o12}$ of Parent results as

$$d_{o12} = j \frac{m_2 t_{p11}^* t_{p12} - m_1 t_{q22}^* t_{q21}^*}{\Im(t_{p11}^* t_{p12}^* t_{q22}^* t_{q21}^*)}$$

Noise parameters can now be calculated from known NCM of Parent
## Allowed Configurations (1...4)

<table>
<thead>
<tr>
<th>Parent</th>
<th>Child#1</th>
<th>Child#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>$d_{Z11} = \langle v_{ZI} v_{ZI}^* \rangle$; $d_{Z22} = \langle v_Z^2 \rangle$</td>
<td>$d_{Y11} = \langle t_{Y1} t_{Y1}^* \rangle$; $d_{Y22} = \langle t_{Y2} t_{Y2}^* \rangle$</td>
</tr>
<tr>
<td></td>
<td>$T_{Z2A} = \begin{bmatrix} 1 &amp; -a_{12} \ 0 &amp; -a_{22} \end{bmatrix}$</td>
<td>$T_{Z2F} = \begin{bmatrix} y_{11} &amp; y_{12} \ y_{21} &amp; y_{22} \end{bmatrix}$</td>
</tr>
<tr>
<td>Z</td>
<td>$d_{Z11} = \langle v_{ZI} v_{ZI}^* \rangle$; $d_{Z22} = \langle v_Z^2 \rangle$</td>
<td>$d_{H11} = \langle v_{HI} v_{HI}^* \rangle$</td>
</tr>
<tr>
<td></td>
<td>$T_{Z2A} = \begin{bmatrix} 1 &amp; -a_{12} \ 0 &amp; -a_{22} \end{bmatrix}$</td>
<td>$T_{Z2H} = \begin{bmatrix} 1 &amp; -h_{12} \ 0 &amp; -h_{22} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>$T_{Z2G} = \begin{bmatrix} -g_{11} &amp; 0 \ -g_{21} &amp; 1 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>$d_{Y11} = \langle t_{Y1} t_{Y1}^* \rangle$; $d_{Y22} = \langle t_{Y2} \rangle$</td>
<td>$d_{Z11} = \langle v_{ZI} v_{ZI}^* \rangle$; $d_{Z22} = \langle v_Z^2 v_{ZI}^* \rangle$</td>
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<td>$T_{Y2A} = \begin{bmatrix} 0 &amp; a_{12} \ 1 &amp; a_{22} \end{bmatrix}$</td>
<td>$T_{Y2Z} = \begin{bmatrix} z_{11} &amp; z_{12} \ z_{21} &amp; z_{22} \end{bmatrix}$</td>
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<td>$d_{G11} = \langle t_{G1} t_{G1}^* \rangle$</td>
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<td>$T_{Y2H} = \begin{bmatrix} -h_{11} &amp; 0 \ -h_{21} &amp; 1 \end{bmatrix}$</td>
<td>$d_{H22} = \langle t_{H2} t_{H2}^* \rangle$</td>
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<td>Y</td>
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# Allowed Configurations (5…8)

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<td><strong>H</strong></td>
<td><strong>G</strong></td>
<td><strong>Z</strong></td>
</tr>
<tr>
<td>(d_{H11} = {v_{H1}^* v_{H1}^*}; \quad d_{H22} = {1, a} )</td>
<td>(d_{G11} = {t_{G1}^* v_{G1}^<em>}; \quad d_{G22} = {v_{G2}^</em> v_{G2}^*} )</td>
<td>(d_{Z11} = {v_{Z1}^* v_{Z1}^*} )</td>
</tr>
<tr>
<td>(T_{H2A} = \begin{bmatrix} 1 &amp; a_{12} \ 0 &amp; a_{22} \end{bmatrix} )</td>
<td>(T_{G2} = \begin{bmatrix} 0 &amp; -z_{12} \ 1 &amp; -z_{22} \end{bmatrix} )</td>
<td>(d_{Y22} = {t_{Y2}^* t_{Y2}^*} )</td>
</tr>
<tr>
<td>(d_{H11} = {v_{H1}^* v_{H1}^*}; \quad d_{H22} = {1, a} )</td>
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<td>(d_{Y22} = {t_{Y2}^* t_{Y2}^*} )</td>
</tr>
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<td>(T_{G2} = \begin{bmatrix} 0 &amp; -z_{12} \ 1 &amp; -z_{22} \end{bmatrix} )</td>
<td>(T_{G2} = \begin{bmatrix} 0 &amp; 0 \ -z_{21} &amp; 1 \end{bmatrix} )</td>
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<tr>
<td><strong>G</strong></td>
<td><strong>H</strong></td>
<td><strong>Z</strong></td>
</tr>
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<td>(d_{G11} = {t_{G1}^* v_{G1}^<em>}; \quad d_{G22} = {v_{G2}^</em> v_{G2}^*} )</td>
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</tr>
</tbody>
</table>
Noise Factor

Noise parameters from chain noise correlation matrix

\[
C_A = \begin{bmatrix}
\langle v_A v_A^* \rangle & \langle v_A t_A^* \rangle \\
\langle t_A v_A^* \rangle & \langle t_A t_A^* \rangle
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} \\
c_{12}^* & c_{22}
\end{bmatrix}
\]

\[
B_{Sopt} = \Im \left( \frac{c_{12}}{c_{11}} \right) ; \quad G_{Sopt} = \sqrt{\frac{c_{22}}{c_{11}} - B_{Sopt}^2} ; \quad R_n = \frac{c_{11}}{2kT} ; \quad F_{min} = 1 + \frac{1}{kT} \left[ \Re(c_{12}) + c_{11}G_{Sopt} \right]
\]

PROCEDURE CODED IN OCEAN-SCRIPT
(CADENCE)
Subcircuit for the DUT
### Spot Noise Report

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ce} )</td>
<td>2</td>
<td>V</td>
</tr>
<tr>
<td>frequency</td>
<td>2e9</td>
<td>Hz</td>
</tr>
<tr>
<td>( T_{amb} )</td>
<td>27</td>
<td>°C</td>
</tr>
</tbody>
</table>

**Global Noise Minimum Conditions**

- \( N_{Fmin} \): 438.7e-3 dB
- \( R_{sopt} \): 5.83e3 ohms
- \( X_{sopt} \): 354.6e-9 H
- \( R_n \): 290.2 ohms
- \( G_{att} \): 19.12 dB
- \( V_{be} \): 833.3e-3 V
- \( I_b \): 224.3e-9 A
- \( I_c \): 77.11e-6 A
- \( mz_0 \): 116.6 (# of instances for \( z_0 \) match)
- \( Ic_{z0} \): 8.991e-3 A
- \( X_{soptz0} \): 3.041e-9 H

**Ohmic Source Minimum Noise Parameters**

- \( N_{Fmin_bcksim} \): 453.3e-3 dB
- \( R_{min} \): 595.3e-3 dB
- \( R_{sopt} \): 2.472e3 ohms
- \( G_{att} \): 19.78 dB
- \( V_{be} \): 862.1e-3 V
- \( I_b \): 545.3e-9 A
- \( I_c \): 194.2e-6 A
- \( mz_0 \): 49.43 (# of instances for \( z_0 \) match)
- \( Ic_{z0} \): 9.599e-3 A

**Ohmic Source Backsimulated Noise Parameters**

- \( N_{Fmin_bcksim} \): 614.8e-3 dB
Min. Noise Figure and Associated Gain
Summary

• Noise parameter computation SPICE simulators
• Cross-correlation term by the involvement of SSPs
• Full noise correlation matrix recovery
• 8 allowed combinations to select from
• Method is applicable to all circuit blocks