“Extraction of RE and its temperature dependence from RF measurements”

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Outline

• requirements against RE extraction
• the $gmx$ method and its refinement
• derivation of the novel $RF$ method
• extraction plots comparing the two approaches
• temperature dependence
• scaling between two emitter lengths
• summary
Preferences

• lack of additional measurement effort
• a transparent extraction theory
• justifiable neglects resulting in a reasonable extraction equation
• proven regression type extraction technique
• usage of unified RF test structures for saving modeling costs

An often used scheme meeting these requirements is the gmx method: extraction from the DC Gummel plot
RE extraction from DC Gummel measurements

In forward active mode

\[ I_c = \frac{c_{10}}{Q_{p,T}} \cdot \exp \left( \frac{V_{biei}}{V_T} \right) \]

\[ V_{biei} = V_{be} - (rb + re) \cdot I_b - re \cdot I_c \approx V_{be} - re \cdot I_c \]

The variation of the weighted charge logarithm is relatively small providing the linear regression [1]

\[ \frac{\partial \ln(I_c)}{\partial V_{be}} = \frac{gmx}{I_c} = -\frac{\partial \ln(Q_{p,T})}{\partial V_{be}} + \frac{1}{V_T} - \frac{re}{V_T} \cdot gmx \]

It is difficult to select the regression interval. Rearranged form

\[ \frac{gmx}{I_c} \approx \frac{1}{V_T} - \frac{re}{V_T} \cdot gmx \]

This should be constant: best satisfied around the extremum
Comparison of the two evaluations

- \( V_{be} \) [V]

- \( re_{-}gmx = 1.04 \Omega \)

Selection of the regression interval is subjective

Minimum location is unique and easy to determine
An expression for the inverse intrinsic transconductance was derived in [2]

\[
\frac{1}{g_m} + re = \frac{\Im(\tilde{h}_{11e})}{\Im(\tilde{h}_{21e})} \tag{1}
\]

Tilde ~ denotes unilateralized (UL) parameters [3]

\[
\begin{align*}
\tilde{y}_{ii} &= y_{ii} + y_{12} \\
\tilde{y}_{21} &= y_{21} - y_{12} \\
\tilde{z}_{ii} &= z_{ii} - z_{12} \\
\tilde{z}_{21} &= z_{21} - z_{12} \\
\tilde{h}_{11} &= \frac{1}{\tilde{y}_{11}} \\
\tilde{h}_{21} &= \frac{\tilde{y}_{21}}{\tilde{y}_{11}} = -\frac{\tilde{z}_{21}}{\tilde{z}_{22}} \\
&\text{for } i = 1, 2
\end{align*}
\]

In UL parameters the transfer branch elements (e.g. parallel capacitances (Y) and series vertical terms like RE (Z) cancel.

An alternative method will be deduced for \( g_m \) making it possible to determine RE from (1)
Simplified equivalent circuit for all present HBT/BJT models
Block#1 parameters

\[ y_{11e}^{(1)} = gpi + s \cdot (cpi + cjc) \]
\[ y_{12e}^{(1)} = -s \cdot cjc \]
\[ y_{21e}^{(1)} = gm - s \cdot cjc \]
\[ y_{22}^{(1)} = go + s \cdot cjc \]

\[ \tilde{y}_{11e}^{(1)} = gpi + s \cdot cpi \]
\[ \tilde{y}_{12e}^{(1)} = -s \cdot cjc \]
\[ \tilde{y}_{21e}^{(1)} = gm \]
\[ \tilde{y}_{22e}^{(1)} = go \]

Block#2 parameters

\[ y_{11e}^{(2)} = \frac{y_{11e}^{(1)} + (re + rci) \Delta Y^{(1)}}{\varepsilon^{(2)}} \]
\[ y_{12e}^{(2)} = \frac{y_{12e}^{(1)} - re \cdot \Delta Y^{(1)}}{\varepsilon^{(2)}} \]
\[ y_{21e}^{(2)} = \frac{y_{21e}^{(1)} - re \cdot \Delta Y^{(1)}}{\varepsilon^{(2)}} \]
\[ y_{22}^{(2)} = \frac{y_{22}^{(1)} + (re + rbi) \Delta Y^{(1)}}{\varepsilon^{(2)}} \]

\[ \tilde{y}_{11e}^{(2)} = \frac{\tilde{y}_{11e}^{(1)} + rci \Delta Y^{(1)}}{\varepsilon^{(2)}} \]
\[ \tilde{y}_{12e}^{(2)} = \frac{\tilde{y}_{12e}^{(1)} + rci \Delta Y^{(1)}}{\varepsilon^{(2)}} \]
\[ \tilde{y}_{21e}^{(2)} = \frac{\tilde{y}_{21e}^{(1)} + rci \Delta Y^{(1)}}{\varepsilon^{(2)}} \]
\[ \tilde{y}_{22e}^{(2)} = \frac{\tilde{y}_{22e}^{(1)} + rci \Delta Y^{(1)}}{\varepsilon^{(2)}} \]

\[ \varepsilon^{(2)} = \Delta Z^{(2)} \cdot \Delta Y^{(1)} = 1 + re \cdot y_{11b}^{(1)} + rbi \cdot y_{11e}^{(1)} + rci \cdot y_{21e}^{(1)} + \Delta r \cdot \Delta Y^{(1)} \]

\[ \Delta r = re \cdot rbi + re \cdot rci + rci \cdot rbi \]
Block#3 parameters

\[ \tilde{y}_{11e}^{(3)} = \tilde{y}_{11e}^{(2)} \]
\[ \tilde{y}_{21e}^{(3)} = \tilde{y}_{21e}^{(2)} \]

Block#4 parameters

\[ \epsilon^{(4)} = \Delta Z^{(4)} \cdot \Delta Y^{(3)} = 1 + rbx \cdot y_{11e}^{(3)} + rcx \cdot y_{22e}^{(3)} + rcx \cdot rbx \cdot \Delta Y^{(3)} \]

\[ \tilde{y}_{11e}^{(4)} = \frac{\tilde{y}_{11e}^{(3)} + rcx \cdot \Delta Y^{(3)}}{\epsilon^{(4)}} \]
\[ \tilde{y}_{11e}^{(4)} = \frac{\tilde{y}_{11e}^{(2)} + rcx \cdot \Delta Y^{(3)}}{\epsilon^{(4)}} = \frac{\tilde{y}_{11e}^{(2)}}{\epsilon^{(4)}} + \frac{rcx}{\Delta Z^{(4)}} = \frac{\tilde{y}_{11e}^{(1)} + rci \cdot \Delta Y^{(1)}}{\epsilon^{(4)} \cdot \epsilon^{(2)}} + rcx \cdot \Delta Y^{(4)} \]

\[ \frac{1}{gm} = \frac{1}{h_{21e}^{(4)}} \cdot \frac{1}{y_{11e}^{(3)}} \cdot \frac{1 - rcx \cdot \tilde{h}_{11e}^{(4)} \cdot \Delta Y^{(4)}}{1 + rci \cdot \tilde{h}_{11e}^{(1)} \cdot \Delta Y^{(1)}} \approx \frac{1}{h_{21e}^{(4)}} \cdot \frac{1}{y_{11e}^{(1)}} \approx \Re \left( \frac{1}{h_{21e}^{(4)}} \right) \cdot \Re \left( \frac{1}{y_{11e}^{(1)}} \right) = \Re \left( \frac{1}{h_{21e}^{(4)}} \right) \cdot \frac{V_T}{I_{bei}} \]

Putting in (1):

\[ re = \frac{\Im (h_{11e})}{\Im (h_{21e})} - \Re \left( \frac{1}{h_{21e}} \right) \cdot \frac{V_T}{I_b} \]

same \( h \) parameters as for \( T_f \)
Le=5um, We=0.27um, Tamb=-40°C to 27°C

- Temperature: -40°C, $\text{re}_\text{rf}=2.74\Omega$, $\text{re}_\text{gm}=2.26\Omega$
- Temperature: 0°C, $\text{re}_\text{rf}=2.85\Omega$, $\text{re}_\text{gm}=2.30\Omega$
- Temperature: -20°C, $\text{re}_\text{rf}=2.83\Omega$, $\text{re}_\text{gm}=2.28\Omega$
- Temperature: 27°C, $\text{re}_\text{rf}=2.88\Omega$, $\text{re}_\text{gm}=2.38\Omega$
Le=5um, We=0.27um, Tamb=50…125°C

- temp= 50°C
  - re rf= 2.94Ω
  - re gm x= 2.44Ω
- temp= 75°C
  - re rf= 2.97Ω
  - re gm x= 2.53Ω
- temp= 100°C
  - re rf= 2.95Ω
  - re gm x= 2.62Ω
- temp= 125°C
  - re rf= 2.99Ω
  - re gm x= 2.74Ω
Le=3um, We=0.27um, Tamb=-40 C° ...

- temp=-40C°
  - re_rf = 5.01Ω
  - re_gmx = 4.17Ω

- temp=0C°
  - re_rf = 5.06Ω
  - re_gmx = 4.27Ω

- temp=27C°
  - re_rf = 4.98Ω
  - re_gmx = 4.37Ω
Le=3um, We=0.27um, Tamb=50...125°C

- Temp = 50°C
  - $V_{CE} = 0.00V$
  - $Re_{rf} = 5.07\,\Omega$
  - $Re_{gm} = 4.50\,\Omega$

- Temp = 75°C
  - $V_{CE} = 0.00V$
  - $Re_{rf} = 5.08\,\Omega$
  - $Re_{gm} = 4.61\,\Omega$

- Temp = 100°C
  - $V_{CE} = 0.00V$
  - $Re_{rf} = 5.02\,\Omega$
  - $Re_{gm} = 4.78\,\Omega$

- Temp = 125°C
  - $V_{CE} = 0.00V$
  - $Re_{rf} = 4.97\,\Omega$
  - $Re_{gm} = 5.00\,\Omega$
Temperature dependence and scaling

Le=5um, We=0.27um

\[ \text{re}_{\text{rf}}_{\text{scaled}} = 2.88 \times 5 = 14.40 \ \Omega \text{um} \]
\[ \text{re}_{\text{gmx}}_{\text{scaled}} = 2.40 \times 5 = 12.00 \ \Omega \text{um} \]

Le=3um, We=0.27um

\[ \text{re}_{\text{rf}}_{\text{scaled}} = 5.04 \times 3 = 15.12 \ \Omega \text{um} \]
\[ \text{re}_{\text{gmx}}_{\text{scaled}} = 4.42 \times 3 = 13.26 \ \Omega \text{um} \]

The gmx method results in smaller RE and a larger temperature dependence.
Summary

• an improvement of the gmx method has been suggested

• the proposed novel RF method is practically free of neglections

• the same RF data is shared what is necessary anyway for the extraction the transit time parameters

• temperature measurements showed slightly positive temperature coefficients

• the gmx method provides consistently smaller RE values
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References

