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Revisiting the Charge Concept in HBT/BJT Models

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Outline

- recalling the junction depletion approximation
- the total charge-total junction voltage relation
- introduction of the *available charge* concept
- overview of the various charge applications
- correcting the temperature dependence of *rbi*
- suggesting alternative MNA charge for model optimization
- summary

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Built in and total voltages

The metallurgical junction is in $x=0$, the doping concentrations in $x<0$ and $x>0$ are of opposite kinds. The electron and hole currents in the structure can be written as [1]

$$I_n = q \cdot A_E \cdot D_n \cdot n \cdot \left(\frac{d \ln(n)}{dx} + \frac{E}{V_T} \right) \qquad I_p = q \cdot A_E \cdot D_p \cdot p \cdot \left(-\frac{d \ln(p)}{dx} + \frac{E}{V_T} \right)$$

Besides the familiar notations A_E is the junction area and V_T is the thermal voltage. When both currents are zero the electric field can be expressed e.g. from the left equation yielding the built in voltage

$$V_B = -\int_{p-side}^{n-side} E(x)dx = V_T \cdot \ln(\overline{n})\Big|_{p-side}^{n-side} = V_T \cdot \ln\left(\frac{\overline{n}_n}{\overline{n}_p} \right) > 0$$

An applied forward voltage $V_a>0$ yields the total voltage across the junction

$$V_{tot} = V_B - V_a$$

The total voltage is always non-negative

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Space charge and junction bias

V_{tot} is maintained by the potential difference over the space charge sustained by the ionized impurity centers. Applying Gauss' theorem within [$-d_L <= \xi <= d_R$] where d_L and d_R stand for the left and right side boundaries of the space charge layer

electric field

$$E(\xi) = \frac{1}{\varepsilon_r \cdot \varepsilon_0} \cdot \int_{-d_L}^{\xi} q \cdot N(x)dx = \frac{1}{\varepsilon_r \cdot \varepsilon_0} \cdot \int_{-d_L}^{\xi} \sigma(x)dx$$

charge balance

$$E(d_R) = 0 \quad \Rightarrow \quad \int_{-d_L}^{d_R} \sigma(x)dx = 0$$

Integrating the negative field by part, the total voltage reads

$$V_{tot} = V_B - V_a = \frac{\text{sgn}}{\varepsilon_r \varepsilon_0} \int_{-d_L}^{d_R} x \cdot \sigma(x)dx \qquad V_B = V_T \frac{\text{sgn}}{\varepsilon_r \varepsilon_0} \int_{-d_{L0}}^{d_{R0}} x \cdot \sigma(x)dx = V_T \cdot \ln\left(\frac{|N(-d_{L0}) \cdot N(d_{R0})|}{n_i^2}\right)$$

The sign is determined by the sequence of the doping types

$$\text{sgn} = \begin{cases} -1 & \text{if} \quad x \cdot \sigma(x) < 0 \\ 1 & \text{if} \quad x \cdot \sigma(x) > 0 \end{cases}$$

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Junction charge increment

Since charge balance unconditionally exists its variation shall also be zero

$$\delta\left(\int_{-d_L}^{d_R}\sigma(x)dx\right) = \int_0^{d_L+\delta_L}\sigma(-x)dx + \int_0^{d_R+\delta_R}\sigma(x)dx = 0$$

Multiplying with the sign *sgn* and the area A_E

$$A_E \cdot \mathrm{sgn} \cdot \delta_L \cdot \sigma(-d_L) + A_E \cdot \mathrm{sgn} \cdot \delta_R \cdot \sigma(d_R) = 0$$

The two terms are the charge increments (of equal magnitudes) on the *left* and *right* sides respectively. The RHS charge is

$$\Delta Q_j = A_E \cdot \mathrm{sgn} \cdot \delta_R \cdot \sigma(d_R)$$

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Junction voltage increment

The variation of the total voltage reads

$$\Delta V_{tot} = \frac{\text{sgn}}{\varepsilon_r \varepsilon_0} \delta \left(\int_{-d_L}^{d_R} x \cdot \sigma(x) dx \right) = \frac{\text{sgn}}{\varepsilon_r \varepsilon_0} \left(- \int_0^{d_L + \delta_L} x \cdot \sigma(-x) dx + \int_0^{d_R + \delta_R} x \cdot \sigma(x) dx \right)$$

Making use of the charge balance variation

$$\Delta V_{tot} = \text{sgn} \cdot \frac{-\delta_L \cdot d_L \cdot \sigma(-d_L) + \delta_R \cdot d_R \cdot \sigma(d_R)}{\varepsilon_r \varepsilon_0} = \frac{\text{sgn} \cdot \delta_R \cdot \sigma(d_R) \cdot (d_L + d_R)}{\varepsilon_r \varepsilon_0} = \frac{d}{A_E \cdot \varepsilon_r \varepsilon_0} \cdot \Delta Q_j$$

Recognizing the junction capacitance:

$$\frac{dQ_j}{dV_{tot}} = C_j(V_{tot})$$

This expression manifests that a larger V_{tot} - increasing the width of the depletion layer - implies a larger (RHS) charge. The rate of increase is C_j.

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Capacitance and actual charge

The semi-empirical capacitance function generally applied in all HBT/BJT device models reads

$$C_j(V_{tot}) = C_{j0}(T_0) \cdot \left(\frac{V_{tot}}{vd_0}\right)^{-z} \qquad V_{tot} = vd - V_a$$

vd and vd_0 are the built-in voltages at arbitrary and nominal temperatures. In transistor modeling the base charge contribution is measured by the charge increment **from** the actual base-side depletion layer position **to** its new position in the base. The actually stored charge is delimited by the metallurgical junction where $C_j = Inf$ or $V_{tot} = 0$. This stored *actual* charge reads

$$Q_{jact}(V_{tot}) = \int_{V_{tot}}^{0} C_j(u)du = -\frac{Q_{jnom}}{1-z} \cdot \left(\frac{V_{tot}}{vd_0}\right)^{1-z} \qquad Q_{jnom} = C_{j0}(T_0) \cdot vd_0$$

The actual charge resides in $[x_j \; d_R]$ when the base is on the RHS

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Application of junction charges in HBT/BJT models

- Description of the Moll-Ross-Gummel charges in AC linked HBT/BJT models
- Modeling the conductivity modulation of the internal base resistance
- Energy storing elements for circuit simulators
- *Recent models apply one single expression for all*

It will be shown that these specific applications benefit from dedicated individual charge formulations

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Moll-Ross-Gummel charges in AC linked approaches

The following corrected MRG charge has been derived in [2]

$$Q_{pT_low} = Q_{p0}(T_0) + \overline{m}_E \cdot Q_{jEnom} \cdot \tilde{\Gamma}_{ET} + \overline{m}_C \cdot Q_{jCnom} \cdot \tilde{\Gamma}_{CT}$$

$$\Gamma_{JT} = \frac{\delta_J}{1 - zJi} \cdot \left[1 - \left(\frac{vdJi}{vdJi_0} \right)^{1-zji} \right] + \Phi_{JT} \qquad \Phi_{JT} = \frac{1}{1-z} \cdot \left[\left(\frac{vdJi}{vdJi_0} \right)^{1-z} - \left(\frac{V_{tot}}{vdJi_0} \right)^{1-z} \right]$$

Formerly it was $\delta_J = 0$ and only Φ_{JT} has taken care of the description of the bias and temperature dependence. The latter is the normalized *applied* charge at the effect of the *applied* voltage

$$\Phi_{JT} = \frac{1}{Q_{jnom}} \int_0^{V_a} C(vd - u)du = -\frac{1}{Q_{jnom}} \int_{vd}^{V_{tot}} C(w)dw = \frac{Q_{jact}(V_{tot}) - Q_{jact}(vd)}{Q_{jnom}}$$

This is referenced to the zero bias actual charge at the same temperature, *used for all purposes in present model variants*

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Base resistance in Hicum/L2

The quiescent part of the stored *minority charge* in the *npn* base reads

$$\tilde{Q}_0 = q \cdot A_E \int_{xdEb0}^{xdCb0} p(x) \cdot dx = q \cdot A_E \int_{xdEb0}^{xdCb0} N(x) \cdot dx$$

xdEb0, xdCb0 are the base-side quiescent depletion boundaries

The conductivity modulation is controlled by the total minority charge between the actual depletion layer boundaries

$$r_i = r_{Bi0} \cdot \frac{\tilde{Q}_0}{\tilde{Q}_0 + \tilde{Q}_{jEtot} + \tilde{Q}_{jCtot} + Q_f}$$

The *total* AC junction related charges are referenced to the **quiescent** depletion boundaries at *zero* bias and *nominal* temperature $v_{totnom}=vd_0$

$$\tilde{Q}_{jtot} = Q_{jact}(V_{tot}) - Q_{jact}(V_{totnom}) = \frac{Q_{jnom}}{1-z} \cdot \left[1 - \left(\frac{V_{tot}}{vd_0} \right)^{1-z} \right]$$

not used in present models

TC of the current *rbi* formulation [3] is deficient

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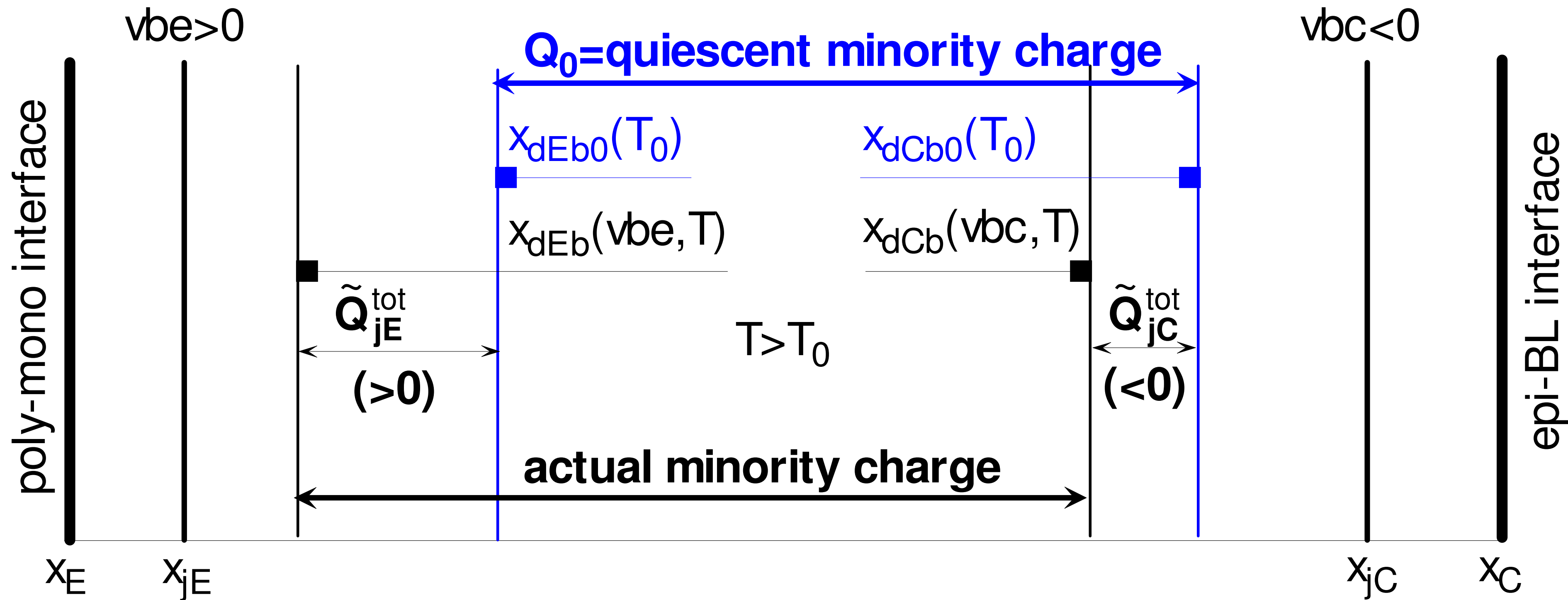
Base resistance in Hicum/L2: interpretation



Existing formulation defines Q_0 as a fraction of the low bias MRG charge with an incorrect temperature dependence. Suggested solution in figure regards Q_0 constant, attributing all bias and temperature dependence to the junction related (total) charges

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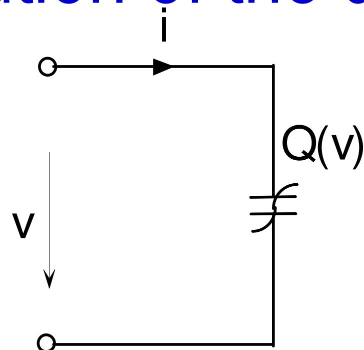
Charges as energy storing elements for MNA

Circuit simulators are using the Modified Nodal Analysis (MNA) for solving the nonlinear network equations. Nonlinear energy storing elements require numerical integration for the solution of the unknown currents and voltages



charge basis

$$i(t) = \frac{dQ(v(t))}{dt}$$

capacitance basis

$$i(t) = C(v(t)) \cdot \frac{dv(t)}{dt} \qquad \Delta Q = C \cdot \Delta v$$

It has been shown by Calahan [4] that an increased stability can be achieved by using charges (fluxes) instead of capacitances (inductors) in the numerical integration schemes

Use of capacitances may violate charge conservation law

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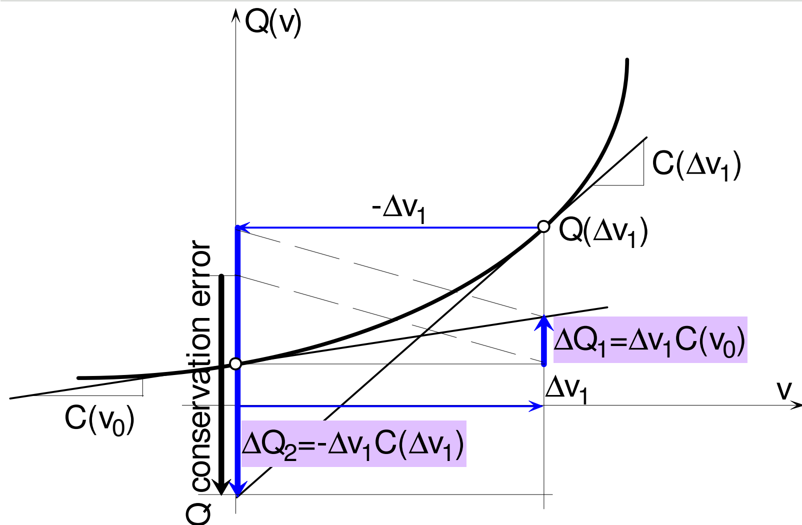
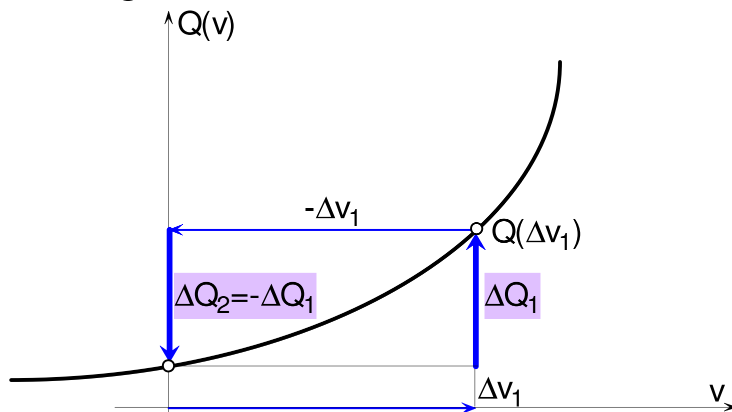
Charges vs. capacitances in MNA



using capacitances the charge difference between timesteps develops along the charge gradients. A closed voltage loop reveals a conservation error



selection of charges as sytem variables inherently gurantees charge conservation

It does not help that the problem exists only at nonlinear charges since it is exactly the case at transistor modeling

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Charges as energy storing elements for MNA, cont.'d

Present models integrate the C(v) function from zero to the actual bias for computing the MNA charge

$$\tilde{Q}_{jMNA}(v) = C_{j0}(T_0) \cdot \int_0^v \left(\frac{vd - u}{vd_0} \right)^{-z} \cdot du = -C_{j0}(T_0) \cdot \int_{vd}^{vtot} \left(\frac{w}{vd_0} \right)^{-z} \cdot dw$$

$$\tilde{Q}_{jMNA}(v) = Q_{jact}(v_{tot}) - Q_{jact}(vd)$$

This is the same charge expression that the one generally used for all purposes in recent models

$$\tilde{Q}_{jMNA} = Q_{jnom} \cdot \tilde{\Phi}_{JT}$$

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Modified charges as energy storing elements for MNA

The first term of the present normalized MNA charge Φ_{JT} is voltage independent hence it has effect neither on the $grad_v(Q)$ Jacobian elements at AC analysis nor on the integration schemes for TR. The preferred alternative is

$$\tilde{Q}_{jMNA} = Q_{jact} = -\frac{Q_{jnom}}{1-z} \cdot \left(\frac{vd - v}{vd_0} \right)^{1-z}$$

This form requires **half the computational effort** than the original Q. The cost of computation reduces by the time spent on approximately evaluating two exponential functions.

Hicum/L2 calls the ddt() operator more than 10 times a computational cycle for nonlinear charges

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Summary of definitions

- ***actual position***: present location of the base-side depletion boundary at a given temperature and bias
- ***actual charge***: charge stored in the interval from the actual position to the metallurgical junction
- ***applied charge***: charge increment from the zero bias position to the applied bias position at the same temperature
- ***total charge***: charge increment from the zero bias, nominal temperature position to the applied bias, applied temperature position

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Overview

Charge used in	Current models	Proposed
AC linked MRG	$\frac{1}{1-z}\cdot\left[\left(\frac{vd}{vd_0}\right)^{1-z}-\left(\frac{v_{tot}}{vd_0}\right)^{1-z}\right]$	$\frac{1}{1-z}\cdot\left[\left(\frac{vd}{vd_0}\right)^{1-z}-\left(\frac{v_{tot}}{vd_0}\right)^{1-z}\right]$
Base resistance	$\frac{1}{1-z}\cdot\left[\left(\frac{vd}{vd_0}\right)^{1-z}-\left(\frac{v_{tot}}{vd_0}\right)^{1-z}\right]$	$\frac{1}{1-z}\cdot\left[1-\left(\frac{v_{tot}}{vd_0}\right)^{1-z}\right]$
MNA	$\frac{1}{1-z}\cdot\left[\left(\frac{vd}{vd_0}\right)^{1-z}-\left(\frac{v_{tot}}{vd_0}\right)^{1-z}\right]$	$-\frac{1}{1-z}\cdot\left(\frac{v_{tot}}{vd_0}\right)^{1-z}$

Formulas are normalized to Q_{jnom}

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Summary

- junction capacitance defined as the derivative of the electrostatic charge w.r.t. the *total voltage*
- modeling applications of the junction charges pinpointed
- present conductivity modulation of *rbi* shown to be deficient
- *actual, applied* and *total* charge concepts introduced
- speed optimization with reduced computational effort and improved *rbi* temperature dependence suggested by using dedicated charge forms

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[2] Z. Huszka, D. Celi and E. Seebacher, „A Novel Low-Bias Charge Concept for HBT/BJT Models Including Heterobandgap and Temperature Effects -- Part I: Theory," IEEE Trans. Electron Dev., submitted

[3] http://www.iee.et.tu-dresden.de/iee/eb/hic_new/hic_doc.html

[4] D. A. Calahan, "Numerical Considerations for Implementation of a Nonlinear Transient Circuit Analysis Program," IEEE Trans. Circuit Theory, CT-18, January 1971, pp.66-73.

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