Review and Advances in Emitter Resistance Extraction Methods

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Outline

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- Conditions required for the derivation of a new method
  - Self-Heating needs to be corrected
  - Transcapacitances impact needs to be estimated
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Motivation

- Emitter resistance determination is a long lasting issue
- Advances in process technologies with $f_{\text{MAX}}$ reaching 500 GHz worsen emitter determination problem
- Extraction of high current densities related parameters becomes extremely complex
  - Performance is reached at the expense of increased current density
  - Combination of several high current physical effects (Kirk effect, parasitic resistance, self-heating, bias dependent Early voltage, B-C barrier…)

Need to determine some of the unknowns separately (e.g., RTH & RE)
Existing Methods

- No specific test structures available for the emitter resistance determination (contrary to the base and extrinsic collector resistance)
- Numerous methods have been developed (based on the analysis of measured data of a given transistor)
  - DC measurements based methods ([KULK1957], [NING1984], [MORI1995], [TRAN1997], …)
  - AC measurements based methods ([KLOO1999], [GOBE1997], [HUSZ2009],…).
What's wrong?

- For very advanced processes, strong self-heating effect brings inaccuracy up to an unacceptable level.
- Especially true for DC methods, exemplified below for the $g_{mx}$ method [HUER2004], [HUSZ2009].
- The $g_{mx}$ method approximates RE by:

$$re \approx \frac{1}{g_{mx}} - \frac{V_T}{I_c}$$

- Where $g_{mx}$ is determined from DC measurements via numerical derivation:

$$g_{mx} \approx \frac{\Delta I_C}{\Delta V_{BE}} \bigg|_{V_{CE}=cst}$$
What's wrong ?

- Let us consider the full derivative:

\[ d_i c = \frac{\partial i}{\partial v_{be}} \bigg|_{v_{ce}} \cdot dv_{be} + \frac{\partial i}{\partial v_{ce}} \bigg|_{v_{be}} \cdot dv_{ce} + \frac{\partial i}{\partial T} \bigg|_{v_{be}} \cdot dT \]

- This clearly states that:

\[ g_{mx} = \frac{\partial i}{\partial v_{be}} \bigg|_{v_{ce}} \]

- In the presence of self-heating, \( dT \neq 0 \), therefore:

\[ g_{mx} \neq \frac{\Delta I_C}{\Delta V_{BE}} \bigg|_{V_{CE}=cst} \]

- \( g_{mx} \) can not be determined from DC data: the method is not valid

- In a similar manner all DC methods are impacted by self-heating
What's wrong?

- What about AC methods?
- Above a few hundred of MHz, the thermal capacitance shunts the thermal resistance: although $\Delta T \neq 0$, there's no small signal variation of $T$ ($dT=0$), therefore

$$
dic = \left. \frac{\partial ic}{\partial vbe} \right|_{vce} \cdot dvbe + \left. \frac{\partial ic}{\partial vce} \right|_{vbe} \cdot dvce + \left. \frac{\partial ic}{\partial T} \right|_{vbe} \cdot dT
$$

Is simplified to:

$$
dic = \left. \frac{\partial ic}{\partial vbe} \right|_{vce} \cdot dvbe + \left. \frac{\partial ic}{\partial vce} \right|_{vbe} \cdot dvce
$$

One can calculate the small signal EC elements (eg, $g_m$) from measured $y$ or $z$ parameters.
What's wrong?

- Therefore AC methods are intrinsically more accurate provided the RE formulation they refer to only contain small signal AC parameters elements (e.g., $y_{21}$, $y_{11}$, $z_{21}$, etc.).

- As it will be seen later this is usually not the case!
  - Gobert & Huszka methods make use of $g_m$ and $g_{be}$ respectively, which need to be calculated from DC data.
  - This leads to the same issue observed for DC methods (dependence on self-heating).
What's wrong?

- Gobert Method [GOBE97] consists in plotting $\text{Re}(Z_{12})$ vs $1/|I_C$
  - Applying a linear regression and taking the y axis intercept gives $\text{RE}$
- Huszka method: intensive (and complex) series of matrix manipulation yields close form expression for RE:

\[
\text{re} \approx \frac{\text{Im}(\tilde{h}_{11})}{\text{Im}(\tilde{h}_{21})} - \text{Re}(\frac{1}{\tilde{h}_{21}}) \frac{V_T}{I_B}
\]

\[
\tilde{h}_{11} = \frac{1}{y_{11} + y_{12}} \quad \tilde{h}_{21} = \frac{y_{21} - y_{12}}{y_{11} + y_{12}}
\]
What's wrong?

- Comparison of $g_{mx}$, Gobert and Huszka method on synthetic data

- Why using synthetic data (eg, simulations with known parameters)?
  - On real measurements there’s no possibility to evaluate the accuracy (errors can/will be compensated by high current parameters)
  - Comparison on synthetic data, gives the lowest error limit achievable by the method for the selected parameter set (needs to be representative of the technology)
  - Can’t be used to prove method’s accuracy
  - Can be used to prove method’s inaccuracy
  - Can switch on/off self-heating in simulations
  - Real method’s accuracy will be higher (depends on measurements, transistor geometry, etc.)
What's wrong?

- Results with self-heating OFF (RE=2Ω black line):

- Without SH $g_{mx}$ method is already quite inaccurate
- Gobert method is fine
- Huszka method almost perfect!
What's wrong?

- Results with self-heating ON (RE=2Ω black line):

- $g_{mx}$ method is 50% off

- Gobert & Huszka methods’ accuracy has been degraded (~10%)
  - Due to the terms $1/I_C$ in the Gobert method and to $V_T/I_B$ in the Huszka method
  - Same issue as for the DC methods
Conditions for a new method

Huszka method is the best, but:

- Derivation still difficult to understand (we need to know limitations)
- Uses a specific base resistance formulation less accurate at low frequency
- Method also less accurate at higher frequencies (reason yet to be understood, may be transcapacitances?)
- Method is valid for frequencies close to 1GHz (may change from one process to another)
- Self-Heating effect MUST be corrected

Let us derive a new formulation to get more insight …
Conditions for a new method

- We start from S-parameters measurements (AC method) @ $V_{BC}=0$:

- Try to find relationships between EC elements and y parameters (or z)
  - Use either simplifications of the EC
  - Or “post de-embedding” of parasitic elements

- Derive an expression for RE

- Correct self-heating

- Compare with Huszka method
New method

HiCUM full EC:

- Colors indicate “post de-embedding steps”
New method

Intrinsic transistor EC (including transcapacitances):
New method

- EC after rigorous “post de-embedding” (need to know all elements)
  - Assumption: rbi merged with rbx
    - @ low frequency cap. splitting is less relevant
    - For the investigated process rbi is “small” compared to rbx and even lower at high current
New method

- EC after simplifications is very similar:
  - Here rbx & rbi are ignored! Extrinsic and intrinsic terms are merged
  - Avalanche & substrate network ignored, rcx ignored
New method

- We can use either the “post de-embedding” EC or the simplified EC to derive our expression of RE
  - This will lead to a “complete method” and a “simplified” method
  - The “complete” method will require the knowledge of some external parasitic elements and will be of lower practical use but it will be useful to evaluate the “simplified” method

- Let us derive two expression for $g_m$ starting from the simplified EC:
New method

- The $[y_i]$ matrix is (w/o RE): $[Y_i] = \begin{bmatrix} Y_{i11} & Y_{i12} \\ Y_{i21} & Y_{i22} \end{bmatrix}$

\[
\begin{align*}
\text{real}(Y_{i11}) &= gjbe + gjbc \\
\text{imag}(Y_{i11}) &= 2\pi f \cdot (CdE_{be} + CdE_{bc} + CdC_{be} + CdC_{bc} + CjE + CjC) \\
\text{real}(Y_{i12}) &= -gjbc \\
\text{imag}(Y_{i12}) &= -2\pi f \cdot (CdE_{bc} + CdC_{bc} + CjC) \\
\text{real}(Y_{i21}) &= gm - gjbc \\
\text{imag}(Y_{i21}) &= -2\pi f \cdot (CdC_{bc} + CdC_{be} + CjC) \\
\text{real}(Y_{i22}) &= g0 + gjbc \\
\text{imag}(Y_{i22}) &= 2\pi f \cdot (CdC_{bc} + CjC)
\end{align*}
\]
New method

- At $V_{BC}=0$ we neglect $g_{jbc}$, therefore:
  \[ \text{real}(Y_{i21}) = gm \]
- from two port theory:
  \[ Y_{21} = -\frac{Z_{21}}{\Delta Z} \]
- Therefore:
  \[ \text{real}(Y_{i21}) = \text{real} \left( -\frac{Z_{i21}}{\Delta Z_i} \right) \]

- Also $[Z_i]$ and $[Z_{RE}]$ are connected in series therefore:
  \[ [Z_x] = [Z_i] + [Z_{RE}] = \begin{bmatrix} Z_{i11} + re_t & Z_{i12} + re_t \\ Z_{i21} + re_t & Z_{i22} + re_t \end{bmatrix} \]
- Therefore:
  \[ \text{real}(Y_{i21}) = \text{real} \left( -\frac{Z_{x21} - re_t}{(Z_{x11} - re_t)(Z_{x22} - re_t) - (Z_{x12} - re_t)(Z_{x21} - re_t)} \right) \]
New method

- This gives the first expression for $g_m$:

$$g_m = \text{real} \left( - \frac{Z_{x21} - re_t}{(Z_{x11} - re_t)(Z_{x22} - re_t) - (Z_{x12} - re_t)(Z_{x21} - re_t)} \right)$$
New method

- Starting from:

\[
[Z_x] = [Z_i] + [Z_{RE}] = \begin{bmatrix}
Z_{i11} + re_t & Z_{i12} + re_t \\
Z_{i21} + re_t & Z_{i22} + re_t
\end{bmatrix}
\]

- Any difference of 2 matrix elements in \([Z_x]\) is equal to the same difference of elements in \([Z_i]\):

\[
Z_{x22} - Z_{x21} = Z_{i22} - Z_{i21} \\
Z_{x12} - Z_{x21} = Z_{i12} - Z_{i21}
\]

- Let’s take the ratio (remove \(\Delta\)):

\[
\frac{Z_{x22} - Z_{x21}}{Z_{x12} - Z_{x21}} = \frac{Z_{i22} - Z_{i21}}{Z_{i12} - Z_{i21}}
\]
New method

- From the Y/Z matrix relations we can easily demonstrate:

\[
\frac{Z_{22} - Z_{21}}{Z_{12} - Z_{21}} = \frac{Y_{11} + Y_{21}}{Y_{21} - Y_{12}}
\]

- Therefore:

\[
\frac{Z_{x22} - Z_{x21}}{Z_{x12} - Z_{x21}} = \frac{Y_{i11} + Y_{i21}}{Y_{i21} - Y_{i12}} = \frac{gbe + gm + j \cdot 2\pi f \cdot (CjE + CdE_{be} + CdC_{be})}{gm + j \cdot 2\pi f \cdot (CdE_{bc} + CdC_{be})}
\]

- In forward mode and before the onset of high current effects:

\[
\frac{Z_{x22} - Z_{x21}}{Z_{x12} - Z_{x21}} \approx \frac{gbe + gm + j \cdot 2\pi f \cdot (CjE + CdE_{be} + CdC_{be})}{gm}
\]
New method

• This gives:

\[ \text{real} \left( \frac{Z_{x22} - Z_{x21}}{Z_{x12} - Z_{x21}} \right) \approx \frac{gbe + gm}{gm} \]

• And finally the second expression for \( g_m \) is:

\[ g_m \approx \frac{gbe}{\text{real} \left( \frac{Z_{x22} - Z_{x21}}{Z_{x12} - Z_{x21}} \right) - 1} \]
New method

Equating the two obtained formulations for $g_m$, and solving for RE gives a 2$^{nd}$ order polynomial from which we can deduce the emitter resistance:

$$re_t = \frac{-b - \sqrt{\Delta}}{2a}$$

with

$$a = G_0 \cdot D_2 - N_2$$
$$b = G_0 \cdot D_1 - N_1$$
$$c = G_0 \cdot D_0 - N_0$$
$$\Delta = b^2 - 4ac$$
$$G_0 = \frac{gbe}{fZ}$$
$$fZ = \text{real}\left(\frac{Z_{x22} - Z_{x21}}{Z_{x12} - Z_{x21}}\right) - 1$$
$$N_0 = -'(real(Z_{x21}) \cdot real(\Delta Z_x) + imag(Z_{x21}) \cdot imag(\Delta Z_x))$$
$$N_1 = real(\Delta Z_x) - real(SZ_x) \cdot real(Z_{x21}) - imag(SZ_x) \cdot imag(Z_{x21})$$
$$N_2 = real(SZ_x)$$
$$D_0 = (real(\Delta Z_x))^2 + (imag(\Delta Z_x))^2$$
$$D_1 = 2 \cdot (real(SZ_x) \cdot real(\Delta Z_x) + imag(SZ_x) \cdot imag(\Delta Z_x))$$
$$D_2 = (real(SZ_x))^2 + (imag(SZ_x))^2$$
$$SZ_x = Z_{x12} + Z_{x21} - Z_{x11} - Z_{x22}$$
$$\Delta Z_x = Z_{x11} \cdot Z_{x22} - Z_{x12} \cdot Z_{x21}$$
New method

- The only remaining unknown in the formulation above is $g_{be}$
- If calculated as $I_B/V_T$ we end up with the same problem as for DC methods
- Therefore, Instead of calculating $V_T$ as:
  \[ V_T = \frac{kT}{q} \]
  Where $T=TNOM$ the measurement temperature.
- We use instead:
  \[ V_T = \frac{k(T_{NOM} + \Delta T)}{q} \]

\[ \Delta T = P \ RTH \approx (I_C V_{CE} + I_B V_{BE}) \ RTH \]
New method

- An additional correction consists in taking into account the non ideality factor (MBE) of the base current:

\[ g_{BE} = \frac{I_B}{MBE \cdot V_T} \]

- MBE needs to be extracted prior to RE extraction which is not always feasible and/or practical
  - The ideal portion of the base current is reduced for advanced processes
Results

- Results for the new method with “post de-embedding” are compared to Huszka method
  - Very Similar results for both methods w/o corrections
  - Self heating correction improves the result
  - Self heating & ideality factor correction gives almost perfect result

- Fundamentally the method is not limited when “rigorous de-embedding” and corrections are applied
Results

- Results for the “simplified” new method are compared to Huszka method
  - w/o any correction Huszka method is better
  - Difference comes from the base resistance simplification
  - Self heating correction improves the result
  - Self heating & ideality factor correction gives excellent result

- Fundamentally the “simplified” method is very efficient when corrections are applied
Results

- The new method has been applied to RE extraction for the last B4T process ($f_T/f_{MAX}$ of approximately 240/350 GHz)
- No specific problems encountered on “real” S parameter measurements
- Results not shown here since it’s not possible to evaluate the emitter resistance extraction accuracy
- Differences are observed between Huszka method and the new method (as expected)

- The new method requires RTH: we have developed a specific extraction method for RTH (not shown here) which can be applied directly to measured data as the first extraction step with a good accuracy (within 10%)
Conclusion

- A new method for RE extraction has been developed.
- The method can be applied with “post de-embedding” techniques or in a simplified form.
- The new method has a different formulation than Huszka method but has very similar results indicating a possible correlation (in the frequency range where Huszka method is valid).
- The derivation of the new method is straight-forward and comprehensive.
- We can apply self-heating and ideality factor corrections to the new method.
- In the presence of self-heating, the new method (with corrections) is more accurate than Huszka method (for synthetic data).
- The “simplified” method with corrections remains more accurate than Huszka method.
More comments for the audience….

- The new method is not considered as the ultimate solution but as a new tool in the toolbox for RE extraction
- Alternative formulations are still under investigation
  - Completely self-heating free method under development (no correction needed)
  - “Zero frequency” S parameters method under development

Note: Huszka method can also be corrected for self-heating in which case it is the most accurate method …
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References


