Extraction of the diffusion voltage and the fixed part of junction capacitances

Zoltan Huszka and Ehrenfried Seebacher

austriamicrosystems AG

24th Bipolar Arbeitkreis (BipAK) Meeting at Infineon
München, Germany, 06 May 2011
• role of the fixed capacitance increases at downscaling
• overview of the known methods to extract $V_d$ and $C_{ox}$
• methods relying in the semiempirical C-V formula are not robust
• a step down to a deeper physical basis is suggested
• regression derived from the Mott-Schottky equation
• method tested on several scaling and T-dependence examples
• summary
Problem

Compact models apply the semiempirical (SE) capacitance-voltage equation

\[ C(V_a) = C_0 + C_j 0 \left( \frac{V_D - V_a}{V_D 0} \right)^{-z} \]

In practice it has model dependent extensions like limiting functions to avoid singularity at \( V_a = V_D \), observing punch-through effect, taking into account charge contributions at high current densities but the basic extraction is always performed on the formula above. The primary problem at parameter extraction is that the fixed capacitance \( C_0 \) (or \( C_{ox} \)) and the diffusion voltage \( V_D \) can not be directly incorporated in a simple linear regression.

Following an overview of the known approaches a novel method will be described that provides the diffusion voltage from the Mott-Schottky equation by linear regression.
Method#1: Multivariable regression & optimization

Set of $V_k$ bias points span a linear regression

Unkowns $C_0$ and $vd$ are determined through minimizing the error of the multivariate linear regression constructed with vectors and matrix

$$a(V_k) = \ln(C(V_k) - C_0)$$

$$a_1(V_k) = 1$$

$$a_2(V_k) = \ln\left(1 - \frac{V_k}{vd}\right)$$

$$A = [a_1 \quad a_2] \quad x = [\ln(C_{j0}); z]$$

$$Ax = a$$

The total least-square error

$$S(C_0, vd) = [Ax - a]^T [Ax - a]$$

can be minimized in terms of $C_0$ and $vd$ by any optimizers e.g. using the standard MATLAB function $fminsearch.m$
Method#1: Multivariable regression & optimization, cont’d

CBEPAR extraction by CBEPAR & vde optimization

\[ C_{je0} = 9.13 \text{fF} \]
\[ z_{je} = 0.151 \]
\[ v_{de} = 0.727 \text{V} \]
\[ \text{CBEPAR} = 3.810 \text{fF} \]

CJC extraction by CBCPAR & vdc optimization

\[ C_{jc0} = 8.20 \text{fF} \]
\[ z_{jc} = 0.313 \]
\[ v_{dc} = 0.716 \text{V} \]
\[ \text{CBCPAR} = 4.059 \text{fF} \]

Each method will be tested by these two measured data
Method#2: Multivariable linear regression on C and Q

Q comes thru integrating C by Simpson’s, \( C_z \) is measurement at zero bias

\[
Q(V_a) = V_a C_0 + \frac{C_{j0} V_d}{1 - z} \left[ 1 - \left( 1 - \frac{V_a}{V_d} \right)^{1-z} \right] = V_a C_0 + \frac{(C_z - C_0) V_d}{1 - z} \left[ 1 - \left( 1 - \frac{V_a}{V_d} \right)^{1-z} \right] \quad C_z = C_0 + C_{j0}
\]

Combining with C provides the 3-variable linear regression

\[
Q(V_a) = V_a C(V_a) \left( \frac{1}{1 - z} \right) - \left[ C(V_a) - C_z \right] \left( \frac{V_d}{1 - z} \right) - V_a \left( C_0 \frac{z}{1 - z} \right)
\]

\[
a(V_k) = Q(V_k)
\]

\[
a_1(V_k) = V_k C(V_k)
\]

\[
a_2(V_k) = -\left( C(V_k) - C_z \right)
\]

\[
a_3(V_k) = -V_k
\]

\[
A = [a_1 \quad a_2 \quad a_3]
\]

\[
A x = a \quad x = \begin{bmatrix} 1/(1 - z); \quad V_d/(1 - z); \quad C_0 z/(1 - z) \end{bmatrix}
\]

No optimization is needed
Method#2: Multivariable linear regression ..., cont'd

CBEPAR extraction by linear regression to Q & C

Linear regression results in negative parameter values at Cje

Cje0 = 9.95 fF
zje = 0.11
vde = 0.58 V
CBEPAR = 22.88 fF
Method#2: Multivariable linear regression ..., cont’d

CBCPAR extraction by linear regression to Q and C

CBCPAR extraction by linear regression to Q and C

Linear regression results look normal at Cjc

\[ C_{jc} = 7.55 \text{fF} \]
\[ z_{jc} = 0.36 \]
\[ V_{dc} = 0.74 \text{V} \]
\[ CBCPAR = 4.72 \text{fF} \]
Method#3: $V_D$ interval exhaustion

C. Raya et. al. proposed a method [1], [2] based on dividing the assumed diffusion voltage interval in subintervals and computing the regression error to the capacitance derivative in the $V_D$ gridpoints. The next interval to divide in subintervals is the one enclosing the point providing the least error. The procedure is stopped when the last $V_D$ interval shrinks to 1mV.

$$f(V) = \frac{dC(V)}{dV} \cdot \left( \frac{dC(0)}{dV} \right)^{-1} = \left( 1 - \frac{V}{vd} \right)^{-z-1}$$

$$\ln(f(V)) = -(z + 1) \cdot \ln\left( 1 - \frac{V}{vd} \right)$$

LHS is measurement, RHS is regressed with $V_D$ in gridpoint

When the optimal $V_D$ is known computation of $C_0$ and $C_{j0}$ is plausible
Method#3: $V_D$ interval exhaustion, cont’d

Emitter capacitance

CBEPAR extraction

CBEPAR extraction by VD exhaustion method

$C_{je0} = 5.84\text{fF}$
$z_{je} = 0.25$
$v_{de} = 0.77\text{V}$
$CBEPAR = 7.10\text{fF}$

Emitter capacitance
Method#3: $V_D$ interval exhaustion, cont’d

CBCPAR extraction

CBCPAR extraction by VD exhaustion method

Collector capacitance

$C_{jc0} = 8.04\, \text{fF}$
$z_{jc} = 0.32$
$v_{dc} = 0.71\, \text{V}$
$\text{CBCPAR} = 4.21\, \text{fF}$
Comparison

Shallow, elongated minima. At fixed $V_D$ each method is robust

Semi-empirical C-V function is too weak to extract 4 parameters from
Method#4: \(vd-C_0\) from capacitance profiling

It has been shown in [3] that the spatial space charge density
represented by the ionized impurity centers implies

**total voltage**

\[
V_{\text{tot}} = V_D - V_a = \frac{\text{sgn}}{\varepsilon_r \varepsilon_0} \int_{-d_L}^{d_R} x \cdot \sigma(x) \, dx
\]

**charge balance**

\[
\int_{-d_L}^{d_R} \sigma(x) \, dx = 0
\]

between the left \(d_L\) and right \(d_R\) edges of the depletion layer.

The sign is determined by the sequence of the doping types

\[
\text{sgn} = \begin{cases} 
-1 & \text{if } x \cdot \sigma(x) < 0 \\
1 & \text{if } x \cdot \sigma(x) > 0 
\end{cases}
\]

It will be shown that the capacitance profiling formula is the direct consequence of this fundamental pair of equations hence providing a possibility for depletion capacitance parameter extraction at arbitrary doping profiles.
Assume a doping sequence $N_D \rightarrow N_A$ that is $\sigma(d_p) = -qN_A(d_p)$ \quad $\sigma(-d_n) = qN_D(-d_n)$

Derivating both sides of the voltage equation w.r.t. $V_{tot}$ provides

$$-1 = \frac{1}{\varepsilon_0} \frac{\partial d_p}{\partial V_{tot}} \cdot \frac{\partial}{\partial d_p} \left[ \int_{-d_n}^{d_p} x \cdot \sigma(x) dx \right] + \frac{1}{\varepsilon_0} \frac{\partial (-d_n)}{\partial V_{tot}} \cdot \frac{\partial}{\partial (-d_n)} \left[ \int_{-d_n}^{d_p} x \cdot \sigma(x) dx \right]$$

$$\frac{1}{\varepsilon_0} \frac{\partial d_p}{\partial V_{tot}} \cdot [d_p \cdot \sigma(d_p)] + \frac{1}{\varepsilon_0} \frac{\partial d_n}{\partial V_{tot}} \cdot [-d_n \cdot \sigma(-d_n)] = - \frac{q}{\varepsilon_0} \frac{\partial d_p}{\partial V_{tot}} \cdot d_p \cdot N_A(d_p) \cdot \left[ 1 + \frac{d_n}{d_p} \cdot \frac{\partial d_n}{\partial d_p} \cdot \frac{N_D(-d_n)}{N_A(d_p)} \right] = -1$$

The variation of the charge balance equation yields

Thus with the total depletion layer width $d = d_n + d_p$ one gets

$$\frac{q}{\varepsilon_0} \frac{\partial d_p}{\partial V_{tot}} \cdot d = \frac{1}{N_A(d_p)}$$

similarly

$$\frac{q}{\varepsilon_0} \frac{\partial d_n}{\partial V_{tot}} \cdot d = \frac{1}{N_D(-d_n)}$$

$$\frac{q}{\varepsilon_0} \frac{\partial d}{\partial V_a} \cdot d = \frac{q}{2\varepsilon_0} \frac{\partial d^2}{\partial V_{tot}} = \frac{1}{N_D(-d_n)} + \frac{1}{N_A(d_p)} = \frac{1}{N}$$
Method#4: capacitance profiling: Mott-Schottky

The junction capacitance \( C_j = \frac{\varepsilon \varepsilon_0 A}{d} \) yields the capacitance profiling equation

\[
\frac{d}{dV_{\text{tot}}} \left( \frac{1}{C_j^2} \right) = \frac{2}{q \varepsilon \varepsilon_0 A^2 \overline{N}}
\]

valid for arbitrary junction forming impurity profiles.

Constant doping yields the **Mott-Schottky** equation [4], [5]

\[
\frac{1}{C_j^2} = \frac{2(V_D - V_a)}{q \varepsilon \varepsilon_0 A^2 \overline{N}}
\]

(since \( C_j(V_{\text{tot}} = 0) = \infty \) the integration constant is zero).

Derivating this function by \( V_a \) and re-using the original expression a linear regression is obtained for \( V_D \) in terms of the the measured capacitance \( C_m \)

\[
C_m + 2V_a \frac{dC_m}{dV_a} = C_0 + 2V_D \frac{dC_m}{dV_a}
\]  \( (1) \)

Despite that the Mott-Schottky is strictly true only for homodoped junctions it had been the starting point of the semi-empirical C-V equation as follows
Method#4: capacitance profiling, SE approximation

A power-function shaped doping profile \( \overline{N} = N_0 \left( \frac{d}{\lambda} \right)^m \) in Mott-Schottky yields at zero bias and nominal temperature:

\[
\frac{1}{C_j^2} = \frac{2(V_D - V_a)}{q\varepsilon\varepsilon_0 A^2 N_0} \left( \frac{d}{\lambda} \right)^{-m}
\]

\[
\left( \frac{C_j}{C_{j0}} \right)^{-2} = \frac{V_D - V_a}{V_{D0}} \left( \frac{d}{d_0} \right)^{-m} = \frac{V_D - V_a}{V_{D0}} \left( \frac{C_j}{C_{j0}} \right)^m
\]

SE formula

\[
C_j = C_0 + C_{j0} \left( \frac{V_D - V_a}{V_{D0}} \right)^{-z}
\]

\[ z = \frac{1}{2 + m} \]

Re-using the original function in the derivative as before results in power-function \( N(x) \)

\[
C_m + \frac{V_a}{z} \frac{dC_m}{dV_a} = C_0 + \frac{V_D}{z} \frac{dC_m}{dV_a}
\]

The reason of the notorious correlation between the diffusion voltage and grading factor is clearly seen: the regression slope is the \( V_D/z \) ratio. The split is relied on the second term of the LHS what is small compared to \( C_m \).
Method#4: capacitance profiling, $V_D$-z correlation

Authors in [6] added pseudo-random stochastic noise with zero mean and 0.5% std. variation to a synthetic C-V data computed from the semi-empirical (SE) formula by $V_D=0.91$, $z=0.46$. One million such perturbed samples have been extracted at room temperature resulting in the inserted correlation plot. Principal distribution axis is a straight $V_D/z$ line as predicted.

Slight departure of a variable from the best fit is compensated by the shift of the other: perfect fits of std. extraction strategies are delusive.
Method #4: reducing the number of freedoms

Mott-Schottky implied the field-proven SE hence (1) on the same basis is also expected to yield correct diffusion voltages at arbitrary doping profiles.

Deviation of extracted diffusion voltage is small even for an E-B like exponential junction: so regressed $V_D$ will be exported to SE extraction.
Method#4: capacitance profiling, $C_{je}$ by imported $V_D$

This and all consequent plots have been obtained by passing $V_D$ from (1) and extracting $C_{j0}$, $C_0$ and $z_j$ from the SE formula.
Method#4: capacitance profiling, Cjc by imported $V_D$

Slight variation of $vdc$ on depletion boundary positions is clearly seen
Method#4: capacitance profiling, Cje scaling

As drawn emitter length (L) and width (W)
Method#4: capacitance profiling, Cjc scaling

More refined scaling techniques have been suggested in [7] (to be tested)
Method#4: T-dependence of the diffusion voltages

Conventional capacitance extraction techniques fail to provide a robust bandgap parameter determination [8]
Summary

• a robust linear regression – derived from the Mott-Schottky equation - have been suggested to extract the diffusion voltage

• the method separates the $v_d$ extraction from the conventional semi-empirical capacitance basis

• extracted $C_{ox}$ and $C_{j0}$ capacitances showed excellent scaling properties over a wide range of $W$ and $L$ variation

• the $v_d$ extraction process utilizes the whole measurement range making bandgap parameters extraction feasible from temperature dependent capacitance data
The authors wish to thank Didier Celi from STMicroelectronicsCrolles, France for the intensive technical discussions and for passing the geometry matrix and temperature measurements.
References


austriamicrosystems - analog experts to help you leap ahead