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Extraction of the diffusion voltage and the fixed part of junction capacitances

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Outline

- role of the fixed capacitance increases at downscaling
- overview of the known methods to extract *vd* and *Cox*
- methods relying in the semiempirical C-V formula are not robust
- a step down to a deeper physical basis is suggested
- regression derived from the Mott-Schottky equation
- method tested on several scaling and T-dependence examples
- summary

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Problem

Compact models apply the semiempirical (SE) capacitance-voltage equation

$$C(V_a) = C_0 + C_{j0}\left(\frac{V_D - V_a}{V_{D0}}\right)^{-z}$$

In practice it has model dependent extensions like limiting functions to avoid singularity at $V_a = V_D$, observing punch-through effect, taking into account charge contributions at high current densities but the basic extraction is always performed on the formula above. The primary problem at parameter extraction is that the fixed capacitance C_0 (or C_{ox}) and the diffusion voltage V_D can not be directly incorporated in a simple linear regression.

Following an overview of the known approaches a novel method will be described that provides the diffusion voltage from the Mott-Schottky equation by linear regression

Method#1: Multivariable regression & optimization

Set of V_k bias points span a linear regression $\ln(C(V_k)-C_0)=\ln(C_{j0})-z\ln\left(1-\frac{V_k}{vd}\right)$

Unknowns C_0 and *vd* are determined through minimizing the error of the multivariate linear regression constructed with vectors and matrix

$$\mathbf{a}(V_k)=\ln(C(V_k)-C_0)$$
$$\mathbf{a}_1(V_k)=1$$
$$\mathbf{a}_2(V_k)=\ln\left(1-\frac{V_k}{vd}\right)$$
$$\mathbf{A}=[\mathbf{a}_1 \quad \mathbf{a}_2] \qquad \mathbf{x}=[\ln(C_{j0});\, z]$$
$$\mathbf{Ax}=\mathbf{a}$$

solution →

$$\mathbf{x}=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{a}$$

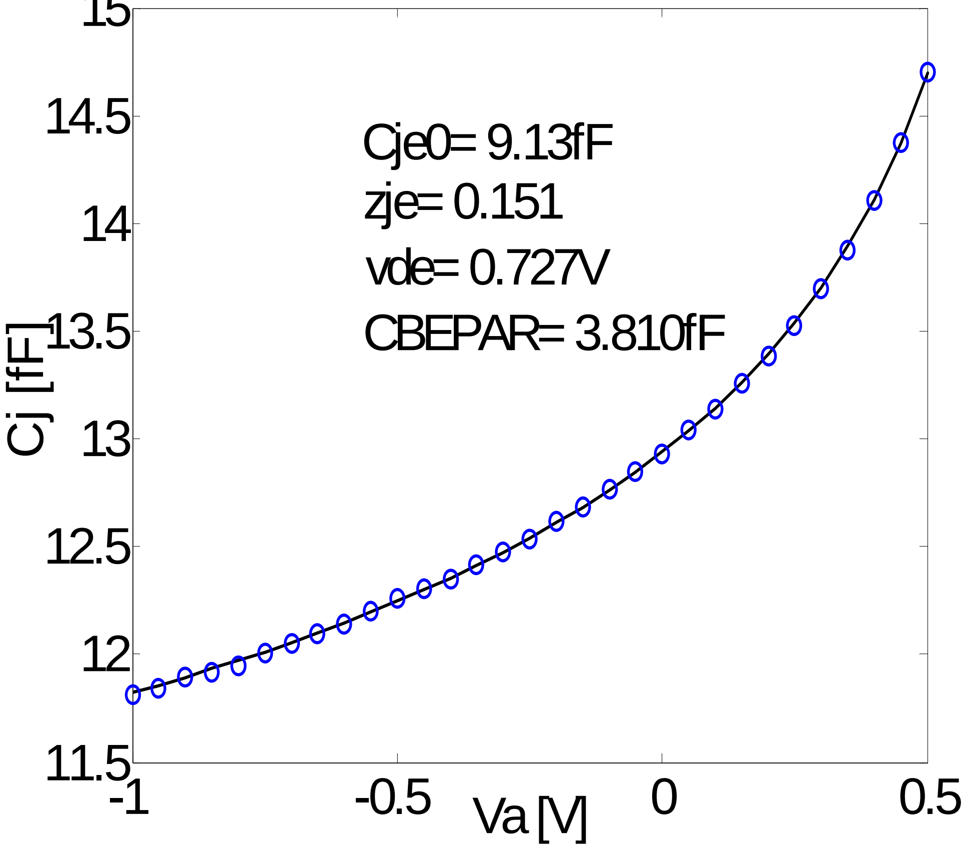
The total least-square error

$$S(C_0,vd)=[\mathbf{Ax}-\mathbf{a}]^T[\mathbf{Ax}-\mathbf{a}]$$

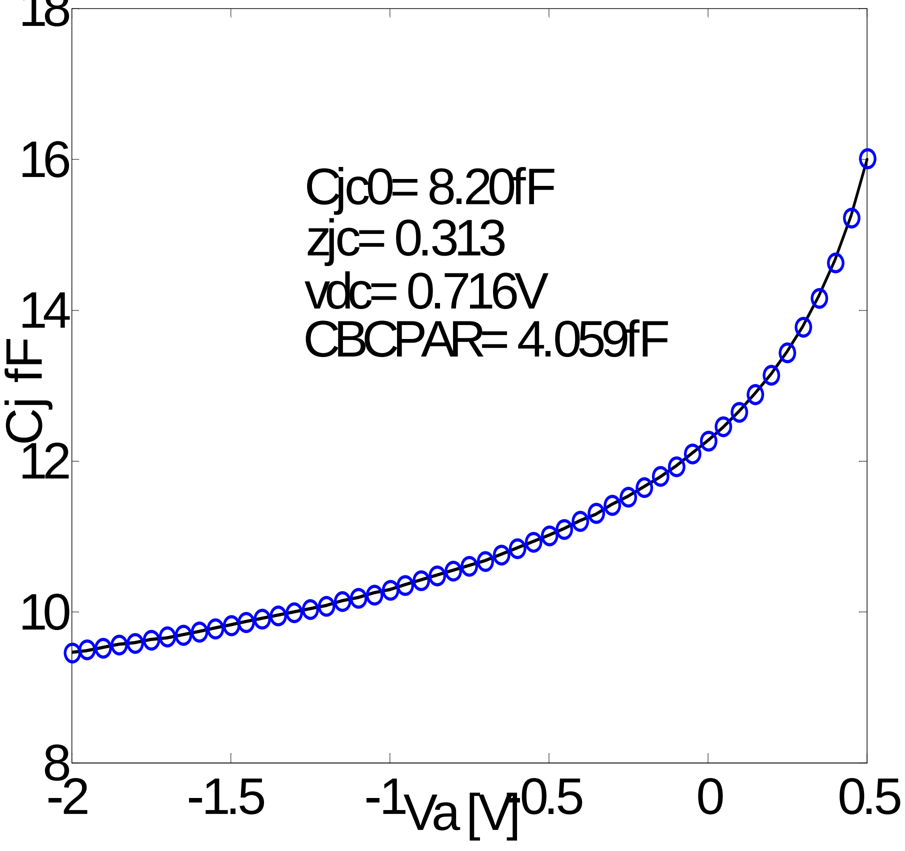
can be minimized in terms of C_0 and *vd* by any optimizers e.g. using the standard MATLAB function *fminsearch.m*

Method#1: Multivariable regression & optimization, cont'd

CBEPAR extraction by CBEPAR & vde optimization



CJC extraction by CBCPAR & vdc optimization



Each method will be tested by these two measured data

Method#2: Multivariable linear regression on C and Q

Q comes thru integrating *C* by Simpson's, C_z is measurement at zero bias

$$Q(V_a) = V_a C_0 + \frac{C_{j0} vd}{1-z}\left[1-\left(1-\frac{V_a}{vd}\right)^{1-z}\right] = V_a C_0 + \frac{(C_z - C_0) vd}{1-z}\left[1-\left(1-\frac{V_a}{vd}\right)^{1-z}\right] \qquad C_z = C_0 + C_{j0}$$

Combining with *C* provides the 3-variable linear regression

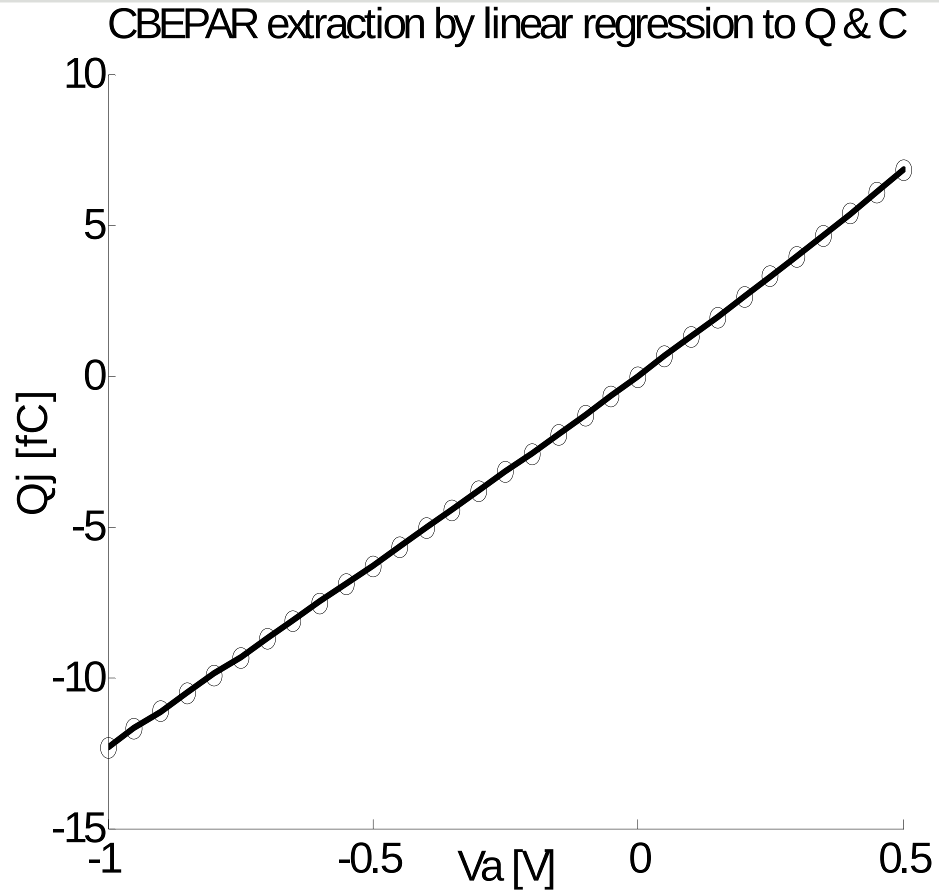
$$Q(V_a) = V_a C(V_a)\left(\frac{1}{1-z}\right) - \left[C(V_a) - C_z\right]\left(\frac{vd}{1-z}\right) - V_a\left(C_0 \frac{z}{1-z}\right)$$

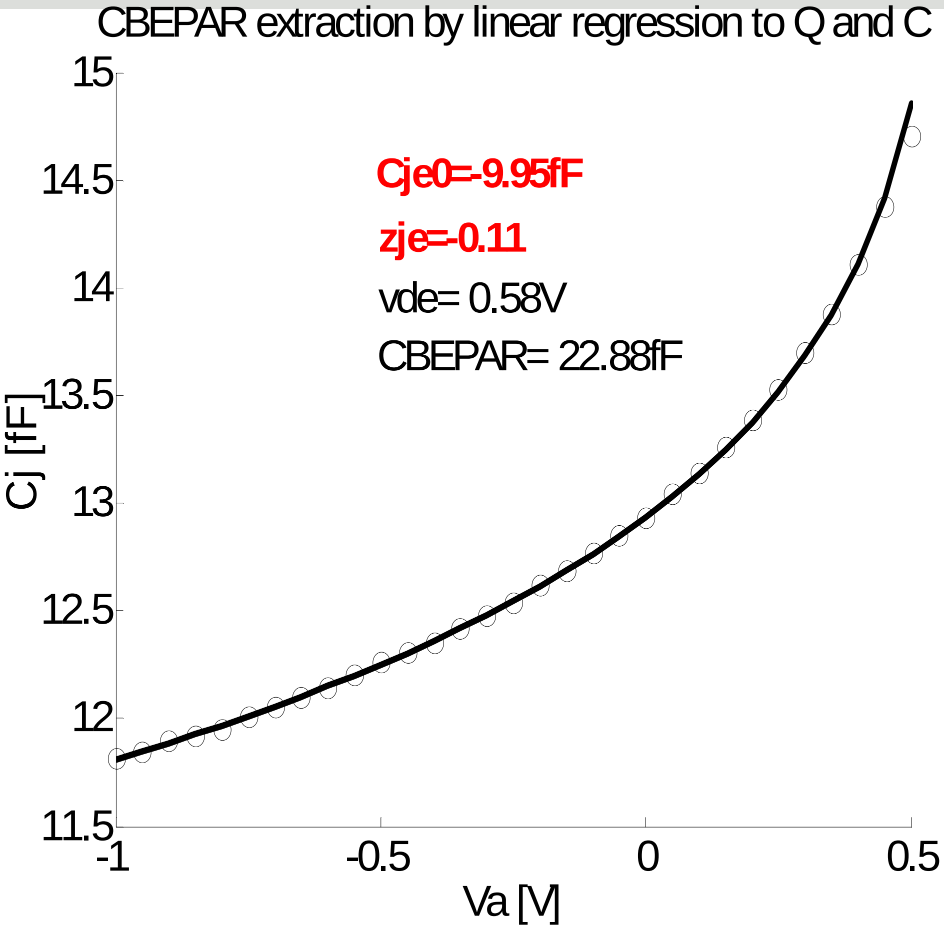
$$\mathbf{a}(V_k) = Q(V_k)$$
$$\mathbf{a}_1(V_k) = V_k C(V_k)$$
$$\mathbf{a}_2(V_k) = -(C(V_k) - C_z)$$
$$\mathbf{a}_3(V_k) = -V_k$$
$$\mathbf{A} = \left[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3\right]$$
$$\mathbf{Ax} = \mathbf{a} \qquad \mathbf{x} = \left[1/(1-z);\, vd/(1-z) \quad C_0 z/(1-z)\right]$$

$$\xrightarrow{\text{solution}} \qquad \mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{a}$$

No optimization is needed

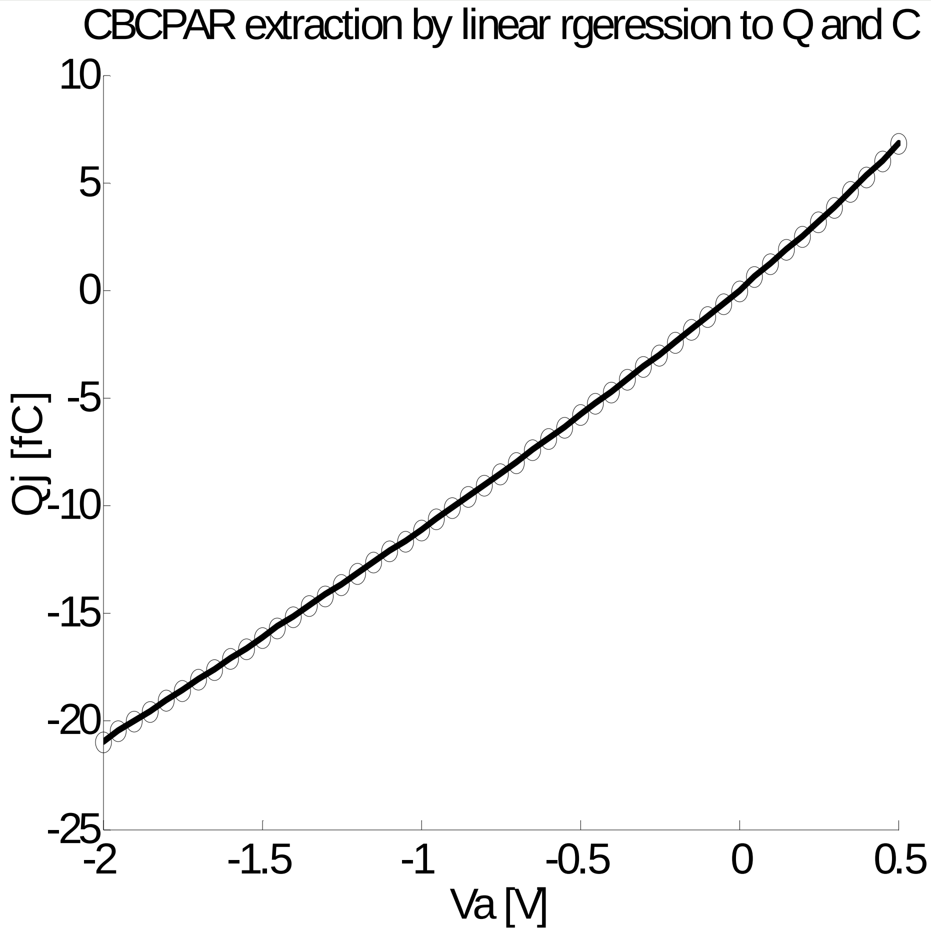
Method#2: Multivariable linear regression …, cont'd

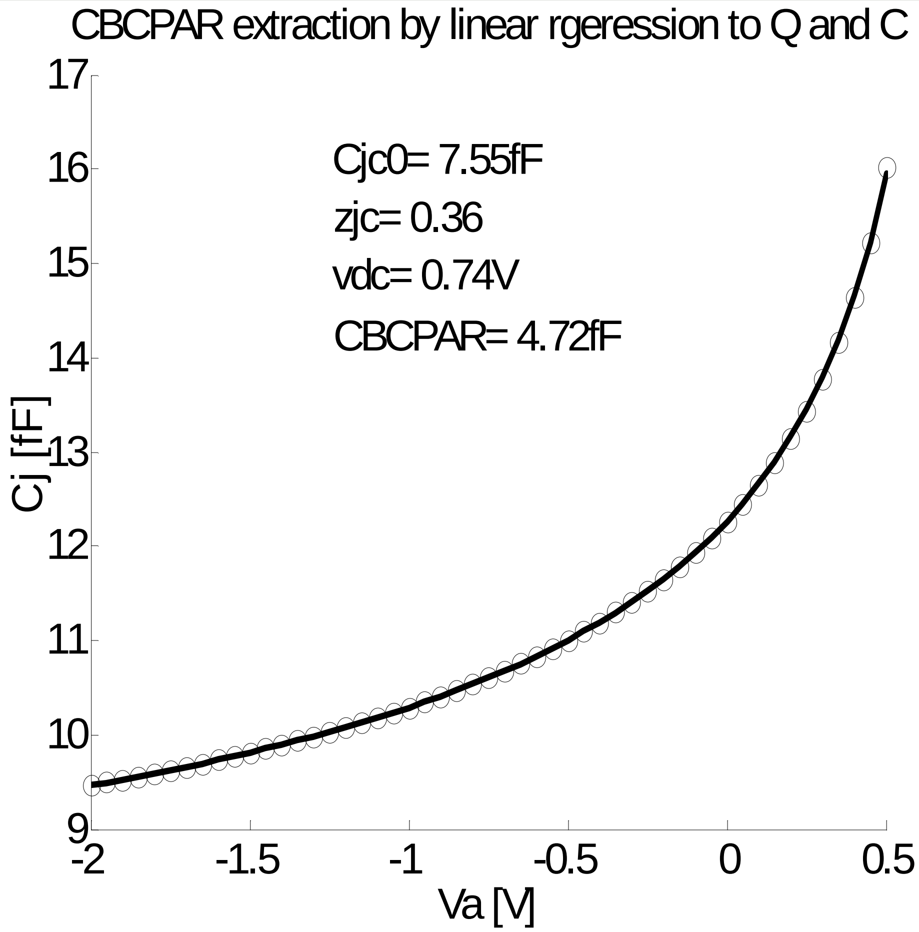




Linear regression results in negative parameter values at Cje

Method#2: Multivariable linear regression …, cont'd





Cjc0= 7.55fF

zjc= 0.36

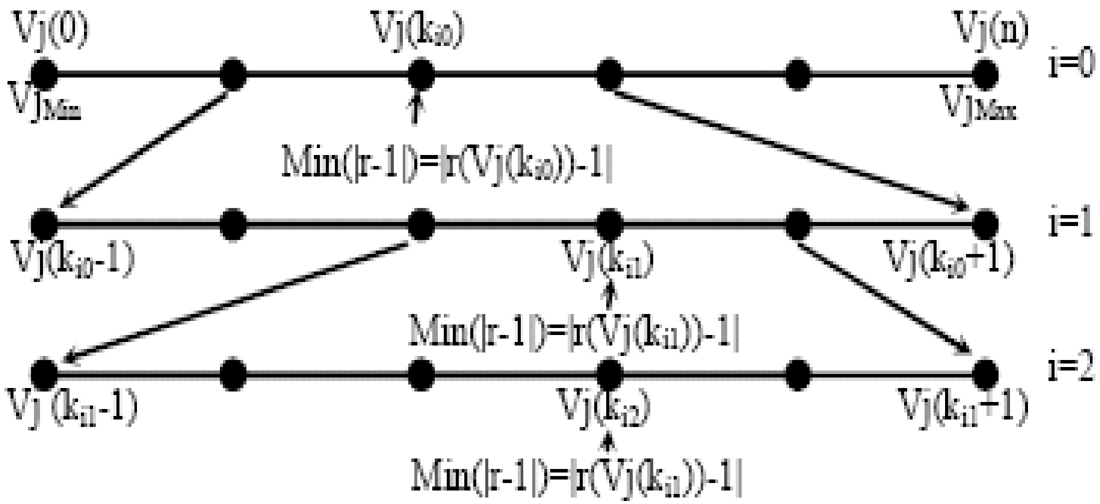
vdc= 0.74V

CBCPAR= 4.72fF

Linear regression results look normal at Cjc

Method#3: V_D interval exhaustion

C. Raya et. al. proposed a method [1], [2] based on dividing the assumed diffusion voltage interval in subintervals and computing the regression error to the capacitance derivative in the V_D gridpoints. The next interval to divide in subintervals is the one enclosing the point providing the least error. The procedure is stopped when the last V_D interval shrinks to 1mV.



$$f(V) = \frac{dC(V)}{dV} \cdot \left(\frac{dC(0)}{dV} \right)^{-1} = \left(1 - \frac{V}{vd} \right)^{-z-1}$$

$$\ln(f(V)) = -(z+1) \cdot \ln\left(1 - \frac{V}{vd} \right)$$

LHS is measurement, RHS is regressed with V_D in gridpoint

When the optimal V_D is known computation of C_0 and C_{j0} is plausible

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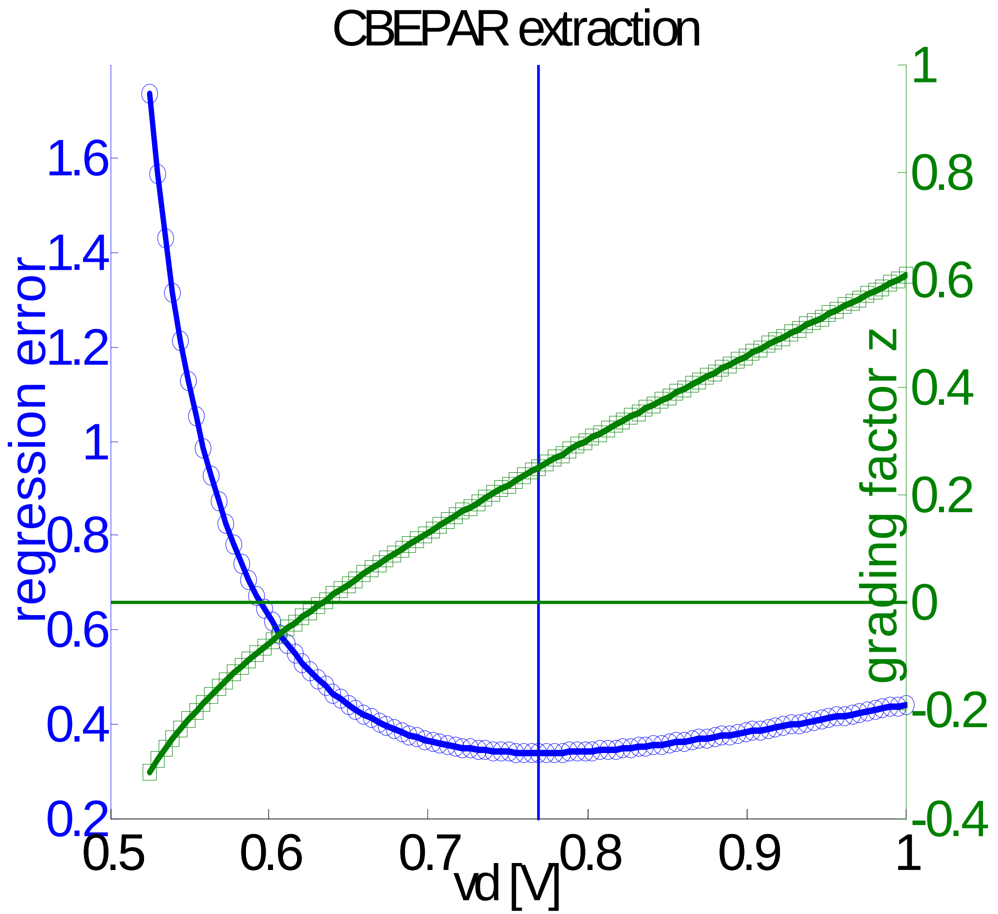
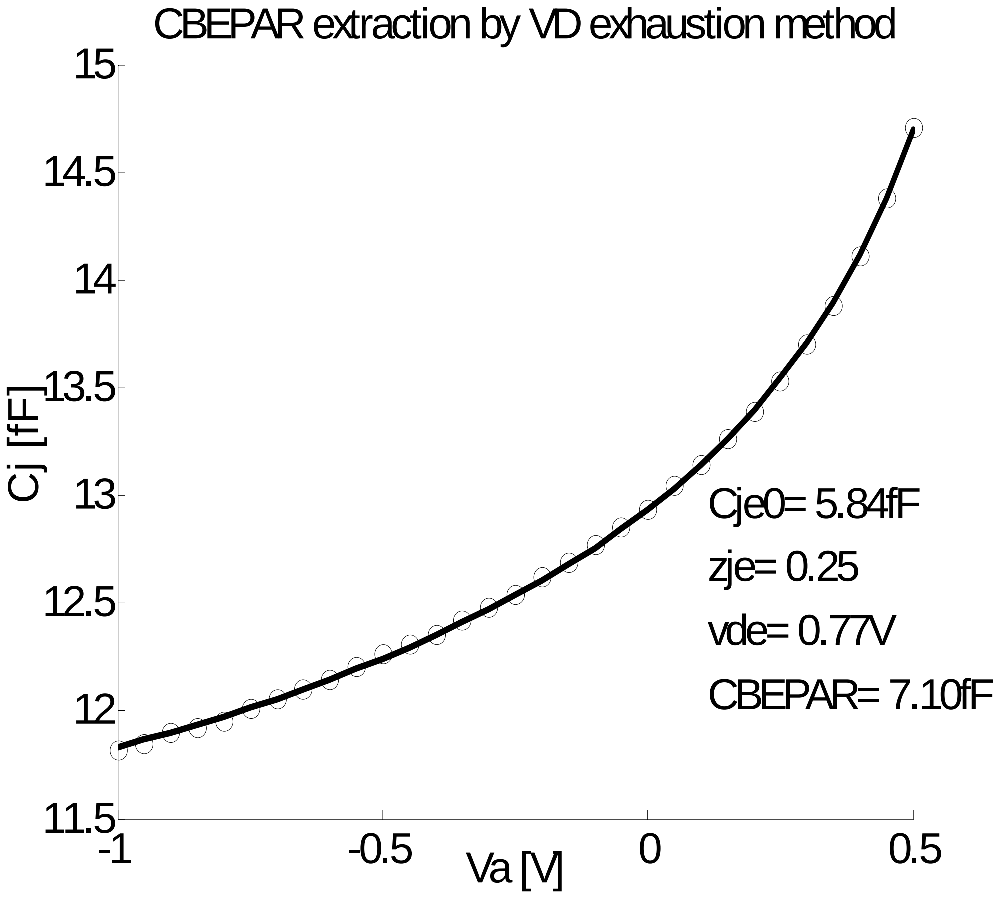
Method#3: V_D interval exhaustion, cont'd

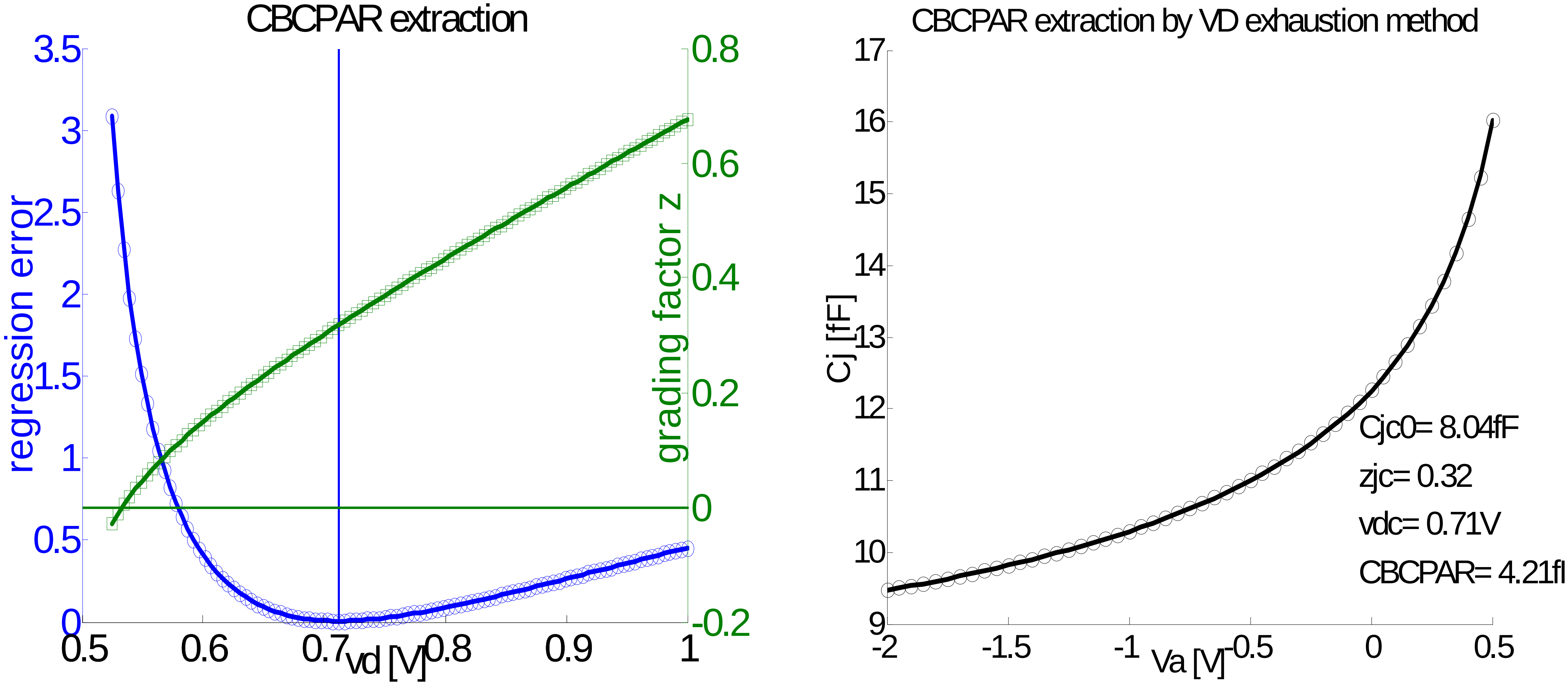




Cje0= 5.84fF
zje= 0.25
vde= 0.77V
CBEPAR= 7.10fF

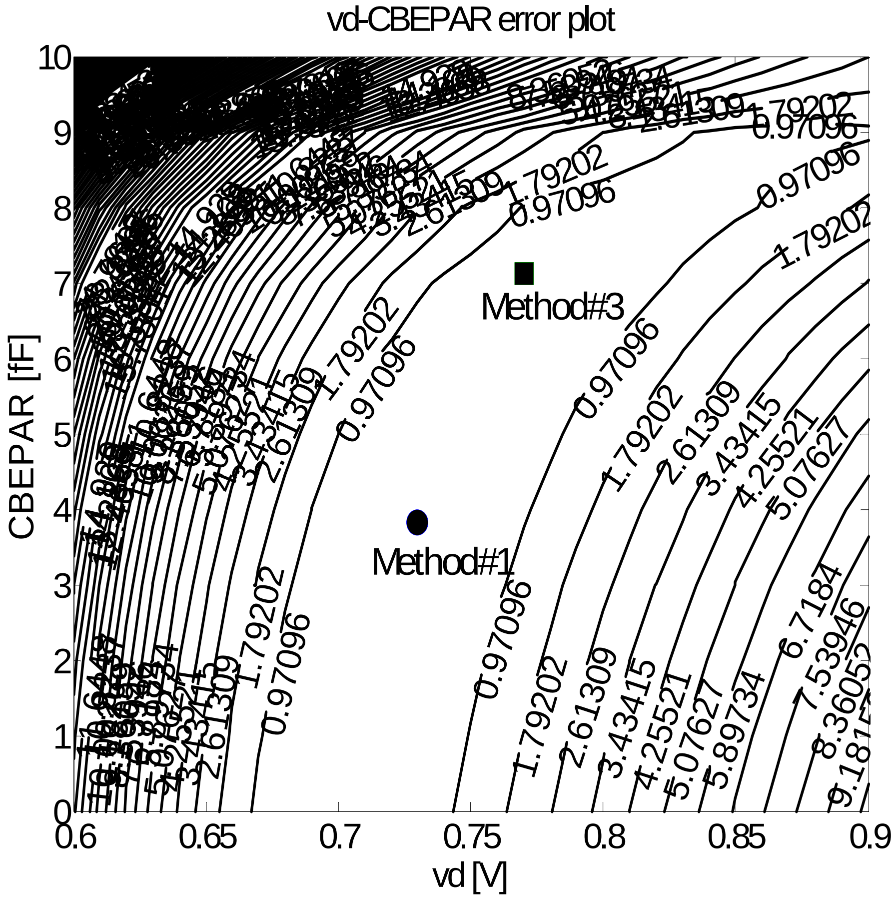
Emitter capacitance

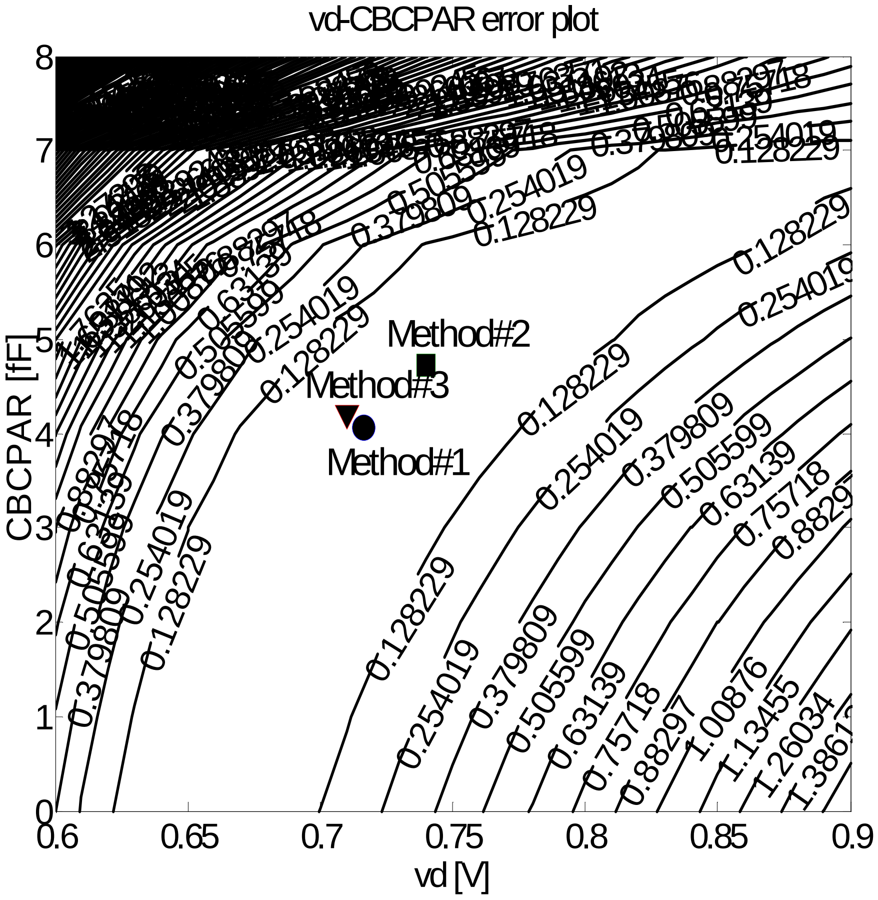
Method#3: V_D interval exhaustion, cont'd



Collector capacitance

Comparison





Shallow, elongated minima. At fixed V_D each method is robust

Semi-empirical C-V function is too weak to extract 4 parameters from

Method#4: *vd*-C_0 from capacitance profiling

It has been shown in [3] that the spatial space charge density $\sigma(x) = q \cdot N(x)$ represented by the ionized impurity centers implies

total voltage

$$V_{tot} = V_D - V_a = \frac{\mathrm{sgn}}{\varepsilon_r \varepsilon_0} \int_{-d_L}^{d_R} x \cdot \sigma(x) dx$$

charge balance

$$\int_{-d_L}^{d_R} \sigma(x) dx = 0$$

between the left d_L and right d_R edges of the depletion layer.

The sign is determined by the sequence of the doping types

$$\mathrm{sgn} = \begin{cases} -1 & \text{if} \quad x \cdot \sigma(x) < 0 \\ 1 & \text{if} \quad x \cdot \sigma(x) > 0 \end{cases}$$

It will be shown that the capacitance profiling formula is the direct consequence of this fundamental pair of equations hence providing a possibility for depletion capacitance parameter extraction at arbitrary doping profiles.

Method#4: capacitance profiling, cont’d

Assume a doping sequence N$_D$ ⟶ N$_A$ that is $\sigma(d_p)=-qN_A(d_p)\quad \sigma(-d_n)=qN_D(-d_n)$

Derivating both sides of the voltage equation w.r.t. *V*$_{tot}$ provides

$$-1=\frac{1}{\varepsilon\varepsilon_0}\frac{\partial d_p}{\partial V_{tot}}\cdot\frac{\partial}{\partial d_p}\left[\int_{-d_n}^{d_p} x\cdot\sigma(x)dx\right]+\frac{1}{\varepsilon\varepsilon_0}\frac{\partial(-d_n)}{\partial V_{tot}}\cdot\frac{\partial}{\partial(-d_n)}\left[\int_{-d_n}^{d_p} x\cdot\sigma(x)dx\right]$$

$$\frac{1}{\varepsilon\varepsilon_0}\frac{\partial d_p}{\partial V_{tot}}\cdot\left[d_p\cdot\sigma(d_p)\right]+\frac{1}{\varepsilon\varepsilon_0}\frac{\partial d_n}{\partial V_{tot}}\cdot\left[-d_n\cdot\sigma(-d_n)\right]=-\frac{q}{\varepsilon\varepsilon_0}\frac{\partial d_p}{\partial V_{tot}}\cdot d_p\cdot N_A(d_p)\cdot\left[1+\frac{d_n}{d_p}\cdot\frac{\partial d_n}{\partial d_p}\cdot\frac{N_D(-d_n)}{N_A(d_p)}\right]=-1$$

The variation of the charge balance equation yields $\dfrac{\delta d_n}{\delta d_p}=\dfrac{N_A(d_p)}{N_D(-d_n)}$

Thus with the total depletion layer width $d=d_n+d_p$ one gets

$$\frac{q}{\varepsilon\varepsilon_0}\frac{\partial d_p}{\partial V_{tot}}\cdot d=\frac{1}{N_A(d_p)}\qquad \text{similarly}\qquad \frac{q}{\varepsilon\varepsilon_0}\frac{\partial d_n}{\partial V_{tot}}\cdot d=\frac{1}{N_D(-d_n)}$$

$$\boxed{\frac{q}{\varepsilon\varepsilon_0}\frac{\partial d}{\partial V_a}\cdot d=\frac{q}{2\varepsilon\varepsilon_0}\frac{\partial d^2}{\partial V_{tot}}=\frac{1}{N_D(-d_n)}+\frac{1}{N_A(d_p)}=\frac{1}{\bar{N}}}$$

Method#4: capacitance profiling: Mott-Schottky

The junction capacitance $C_j = \frac{\varepsilon\varepsilon_0 A}{d}$ yields the capacitance profiling equation

$$\boxed{\frac{\partial}{\partial V_{tot}}\left(\frac{1}{C_j^2}\right) = \frac{2}{q\varepsilon\varepsilon_0 A^2 \overline{N}}}$$

valid for arbitrary junction forming impurity profiles

Constant doping yields the **Mott-Schottky** equation [4], [5] (since $C_j(V_{tot}=0)=\infty$ the integration constant is zero)

$$\boxed{\frac{1}{C_j^2} = \frac{2(V_D - V_a)}{q\varepsilon\varepsilon_0 A^2 \overline{N}}}$$

Derivating this function by V_a and re-using the original expression a linear regression is obtained for V_D in terms of the the measured capacitance C_m

$$C_m + 2V_a \frac{dC_m}{dV_a} = C_0 + 2V_D \frac{dC_m}{dV_a} \qquad (1)$$

Despite that the Mott-Schottky is strictly true only for homodoped junctions it had been the starting point of the semi-empirical C-V equation as follows

Method#4: capacitance profiling, SE approximation

A power-function shaped doping profile $\overline{N} = N_0\left(\frac{d}{\lambda}\right)^m$ in Mott-Schottky yields

$\frac{1}{C_j^2} = \frac{2(V_D - V_a)}{q\varepsilon\varepsilon_0 A^2 N_0}\left(\frac{d}{\lambda}\right)^{-m}$ at zero bias and nominal temperature: $\frac{1}{C_{j0}^2} = \frac{2V_{D0}}{q\varepsilon\varepsilon_0 A^2 N_0}\left(\frac{d_0}{\lambda}\right)^{-m}$

$\left(\frac{C_j}{C_{j0}}\right)^{-2} = \frac{V_D - V_a}{V_{D0}}\left(\frac{d}{d_0}\right)^{-m} = \frac{V_D - V_a}{V_{D0}}\left(\frac{C_j}{C_{j0}}\right)^{m}$ SE formula → $C_j = C_0 + C_{j0}\left(\frac{V_D - V_a}{V_{D0}}\right)^{-z}$ $z = \frac{1}{2+m}$

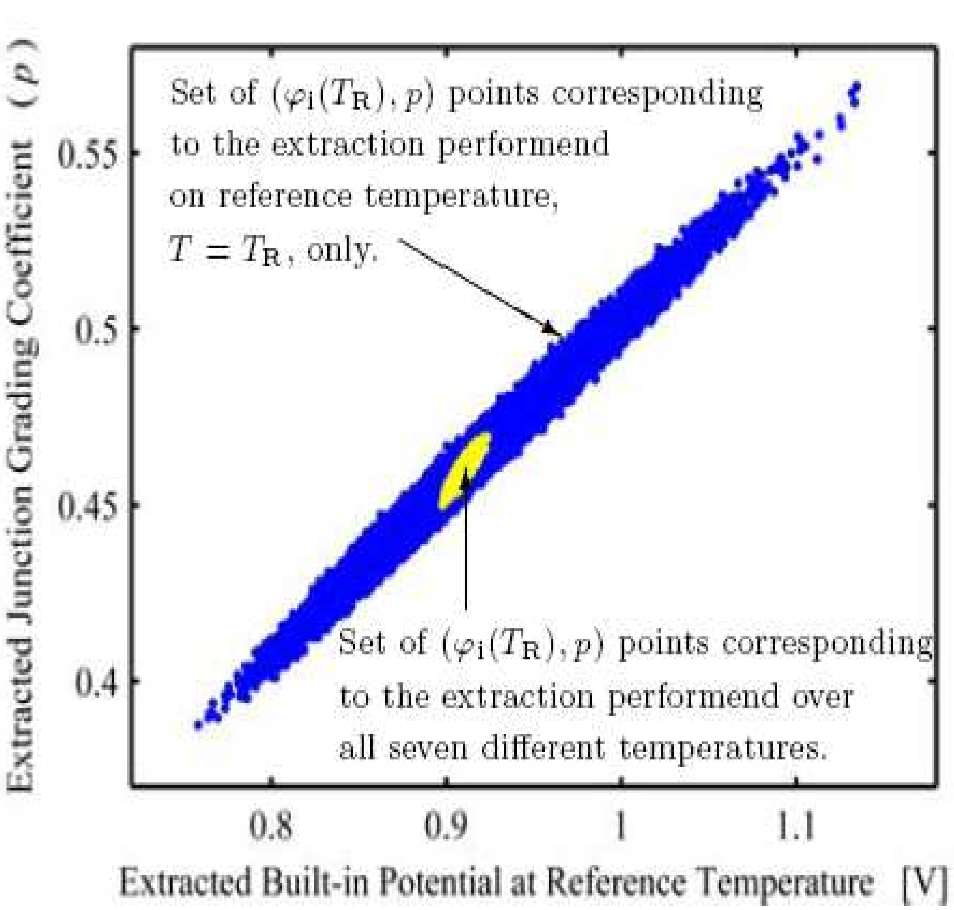
Re-using the original function in the derivative as before results in

power-function N(x)

$$\boxed{C_m + \frac{V_a}{z}\frac{dC_m}{dV_a} = C_0 + \frac{V_D}{z}\frac{dC_m}{dV_a}}$$

The reason of the notorious correlation between the diffusion voltage and grading factor is clearly seen: the regression slope is the V_D/z ratio. The split is relied on the second term of the LHS what is small compared to C_m

Method#4: capacitance profiling, V_D-z correlation

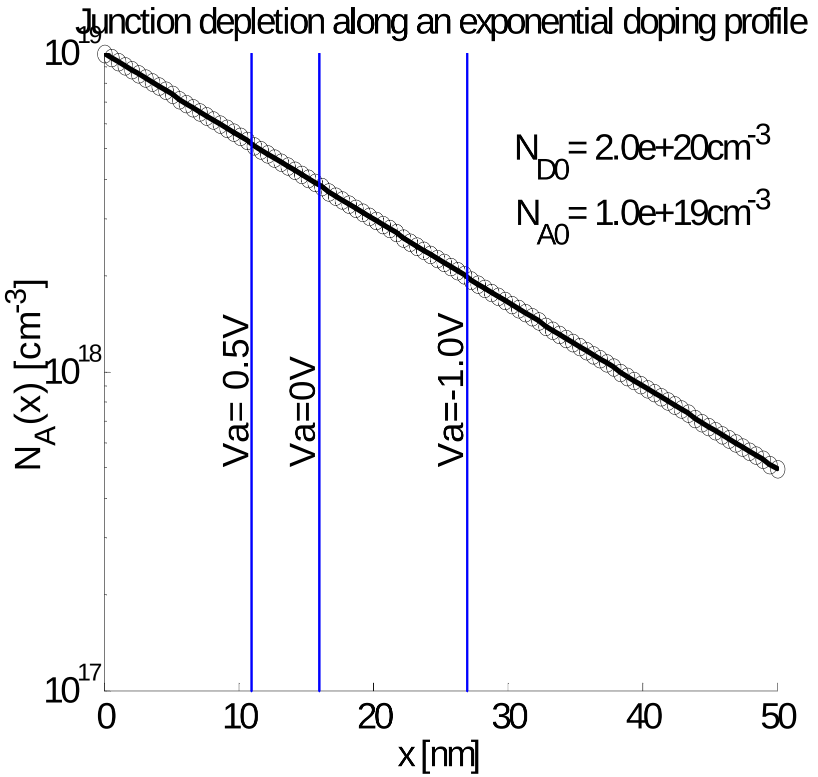


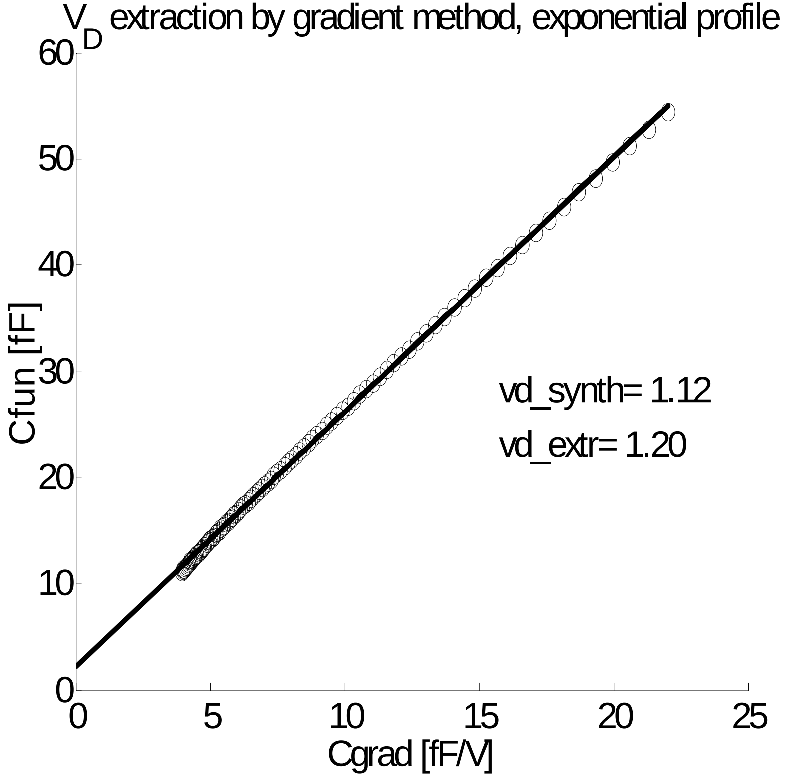
Authors in [6] added pseudo-random stochastic noise with zero mean and 0.5% std. variation to a synthetic C-V data computed from the semi-empirical (SE) formula by $V_D=0.91$, $z=0.46$. One million such perturbed samples have been extracted at room temperature resulting in the inserted correlation plot. Principal distribution axis is a straight V_D/z line as predicted.

Slight departure of a variable from the best fit is compensated by the shift of the other: perfect fits of std. extraction strategies are delusive

Method#4: reducing the number of freedoms

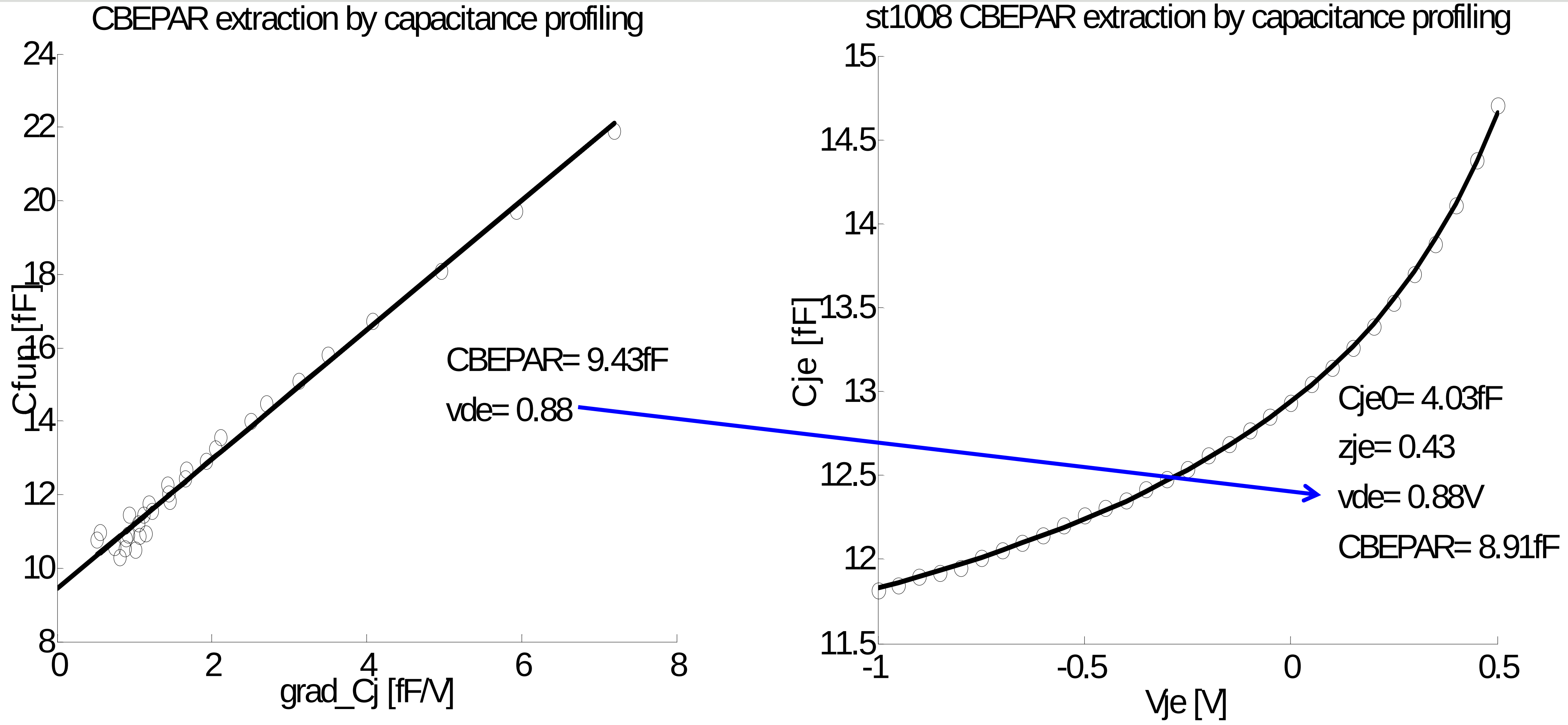
Mott-Schottky implied the field-proven SE hence (1) on the same basis is also expected to yield correct diffusion voltages at *arbitrary* doping profiles


Junction depletion along an exponential doping profile


V_D extraction by gradient method, exponential profile

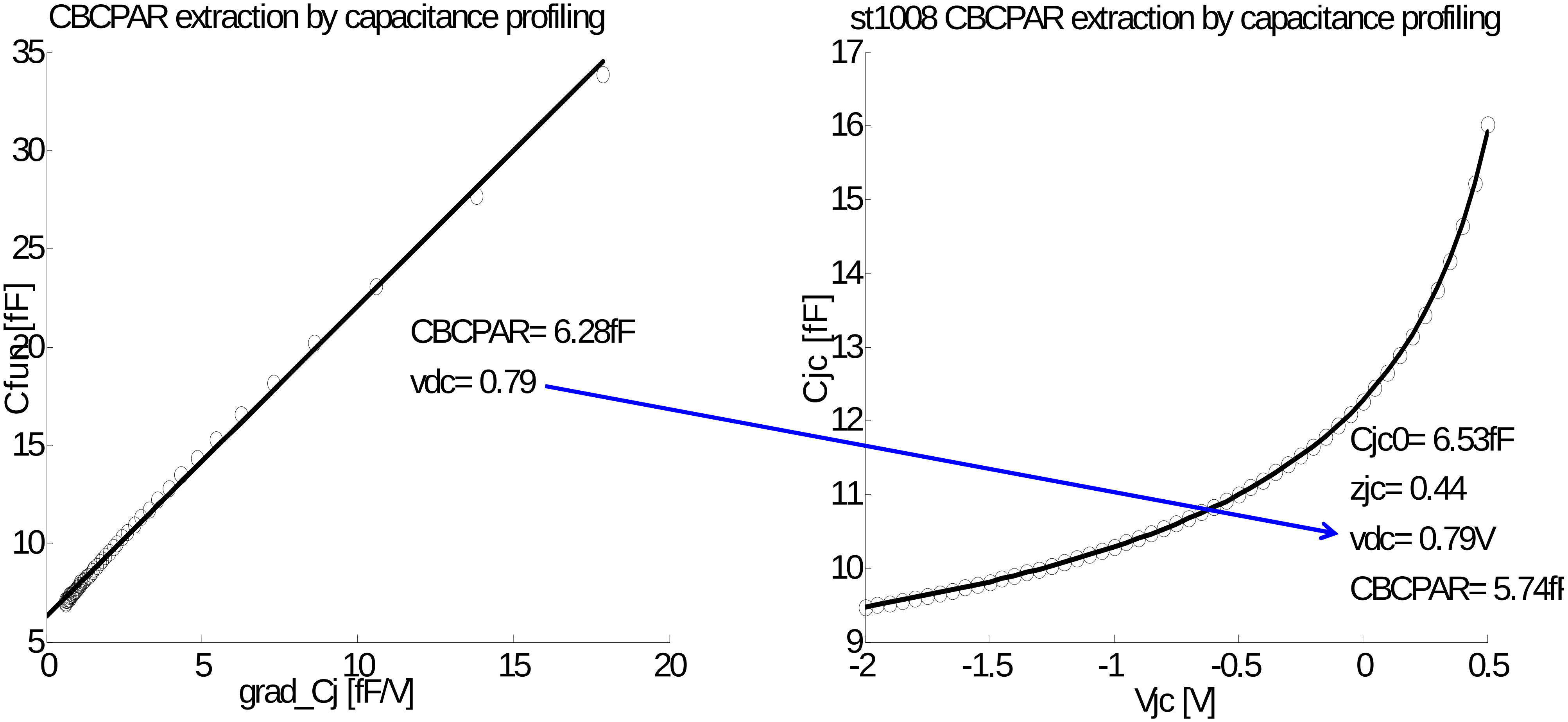
Deviation of extracted diffusion voltage is small even for an E-B like exponential junction: so regressed V_D will be exported to SE extraction

Method#4: capacitance profiling, Cje by imported V_D



This and all consequent plots have been obtained by passing V_D from (1) and extracting C_{j0}, C_0 and z_j from the SE formula

Method#4: capacitance profiling, Cjc by imported V_D



CBCPAR extraction by capacitance profiling

st1008 CBCPAR extraction by capacitance profiling

CBCPAR= 6.28fF

vdc= 0.79

Cjc0= 6.53fF

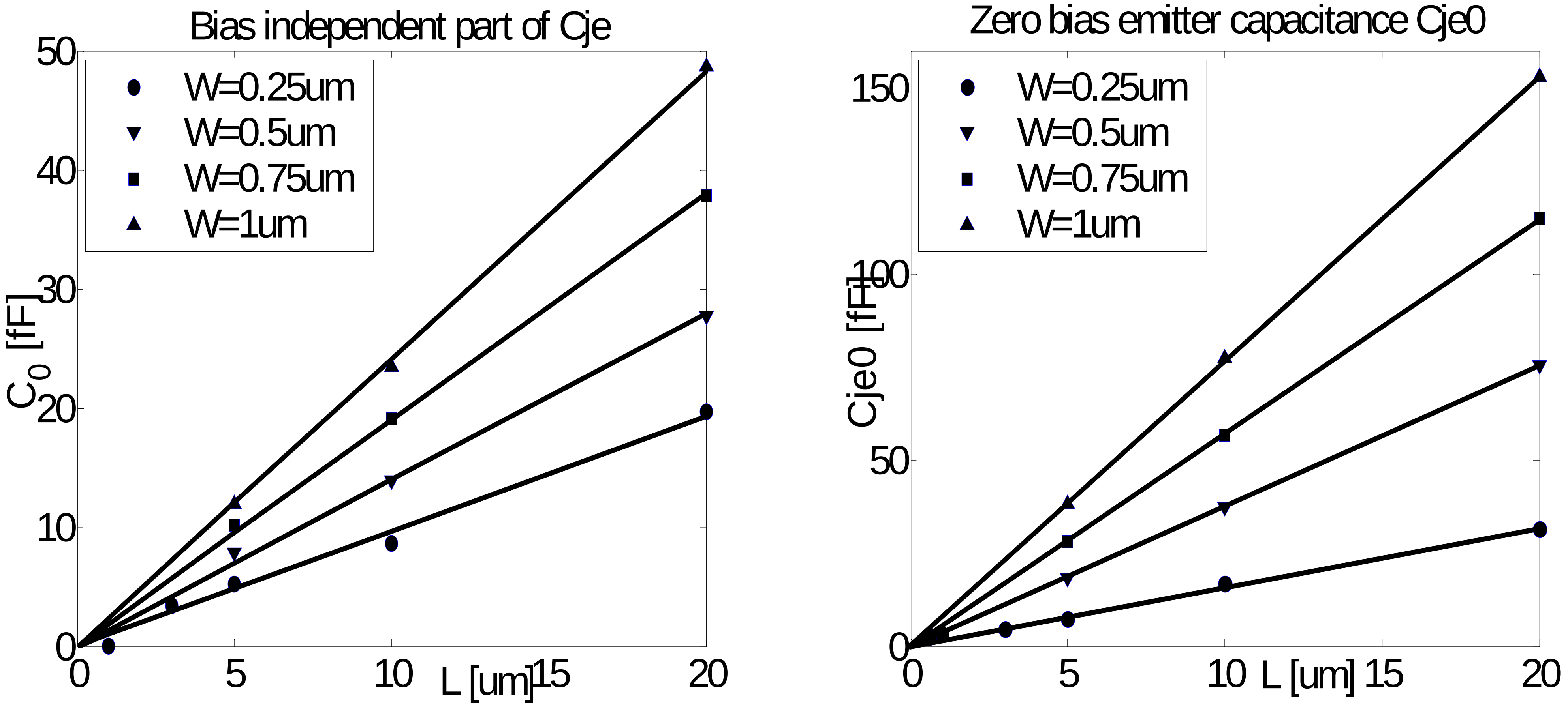
zjc= 0.44

vdc= 0.79V

CBCPAR= 5.74fF

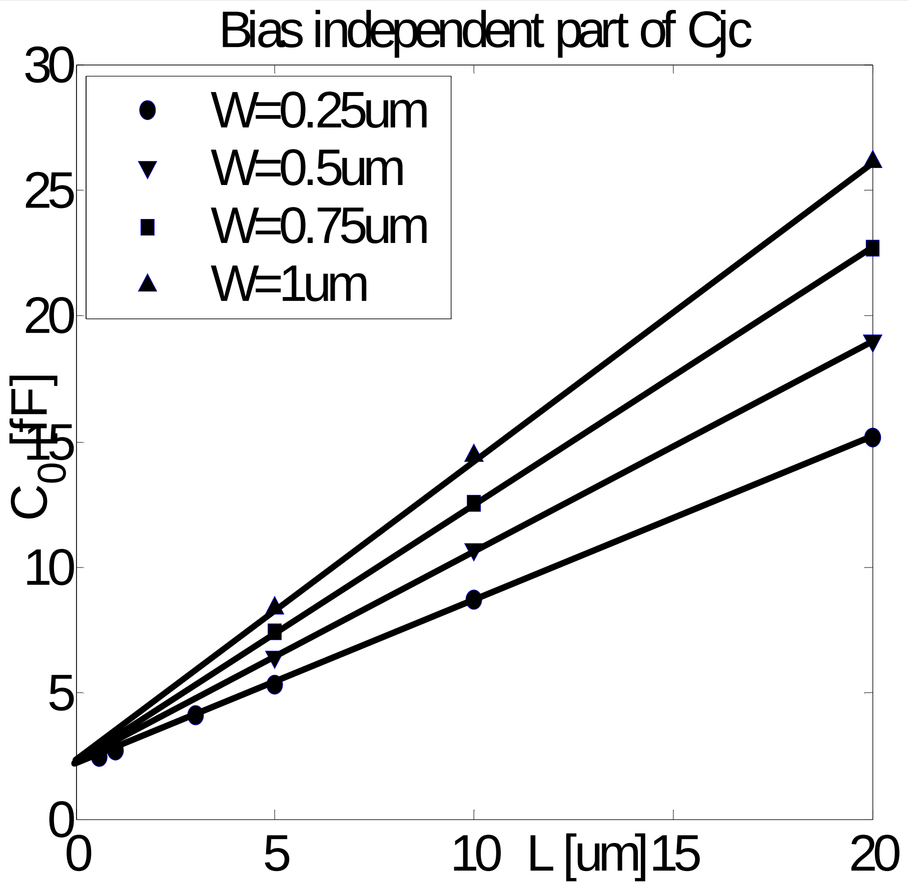
Slight variation of *vdc* on depletion boundary positions is clearly seen

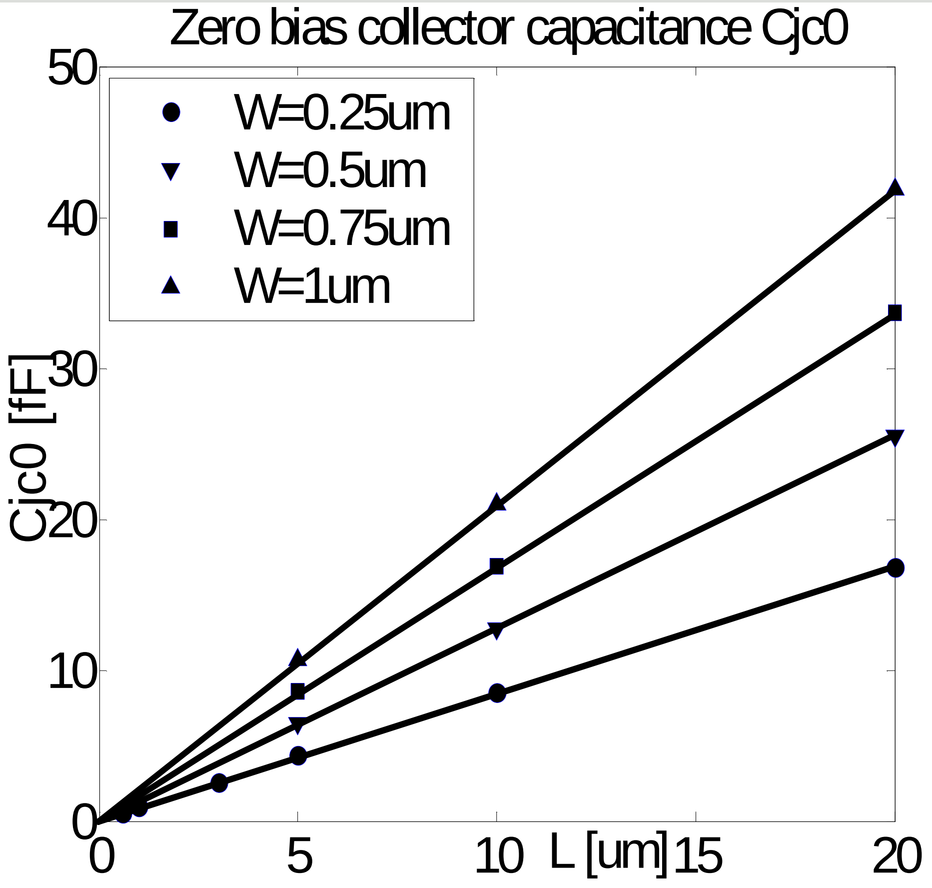
Method#4: capacitance profiling, Cje scaling



As drawn emitter length (L) and width (W)

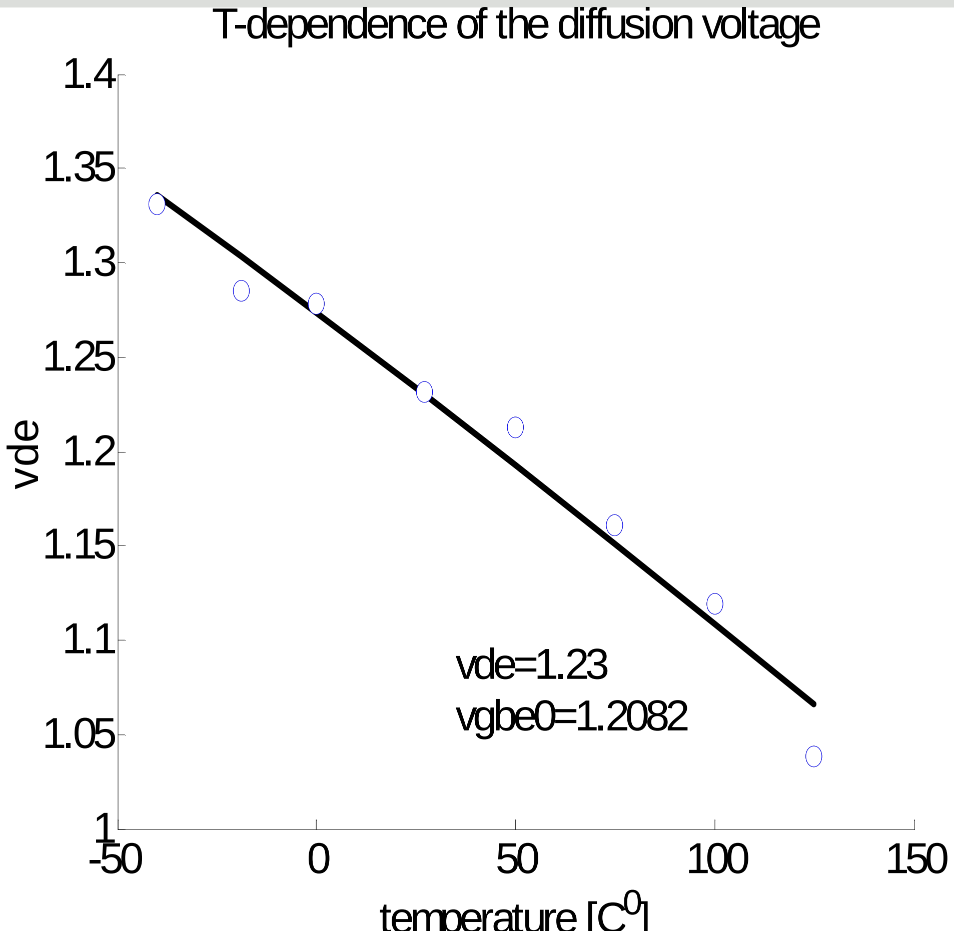
Method#4: capacitance profiling, Cjc scaling

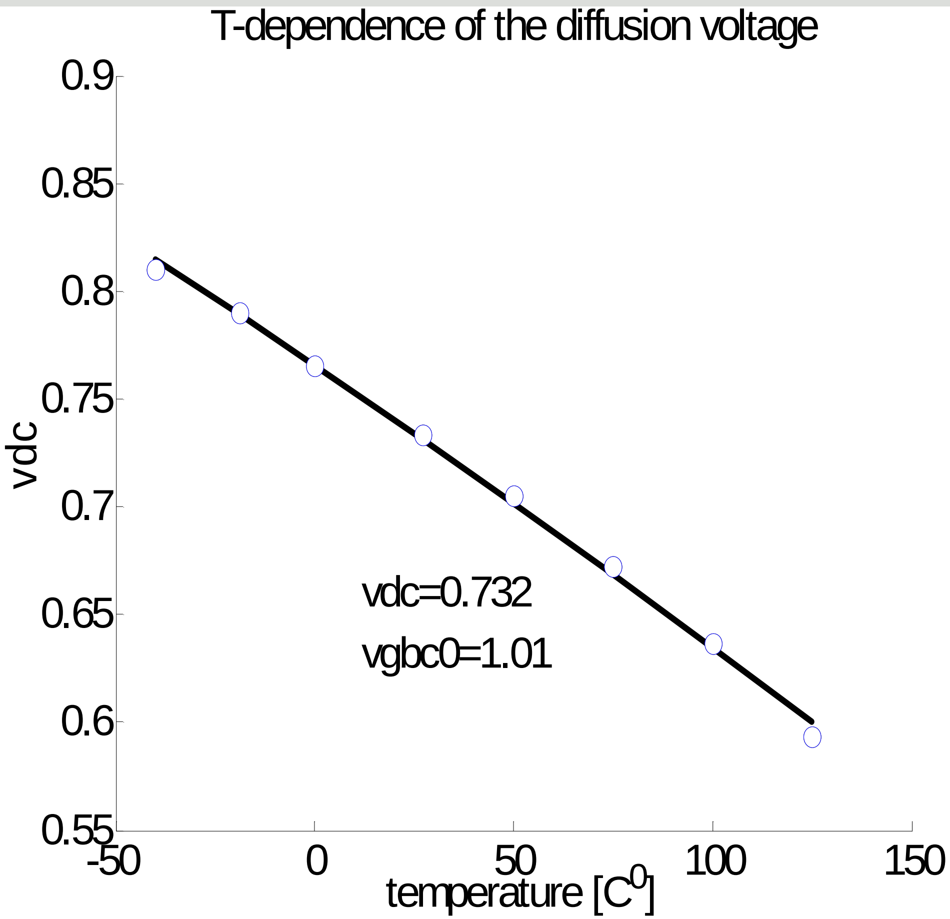




More refined scaling techniques have been suggested in [7] (to be tested)

Method#4: T-dependence of the diffusion voltages





Conventional capacitance extraction techniques fail to provide a robust bandgap parameter determination [8]

Summary

- a robust linear regression – derived from the Mott-Schottky equation - have been suggested to extract the diffusion voltage
- the method separates the *vd* extraction from the conventional semi-empirical capacitance basis
- extracted *Cox* and *Cj0* capacitances showed excellent scaling properties over a wide range of *W* and *L* variation
- the *vd* extraction process utilizes the whole measurement range making bandgap parameters extraction feasible from temperature dependent capacitance data

Acknowledgment

The authors wish to thank **Didier Celi** from STMicroelectronicsCrolles, France for the intensive technical discussions and for passing the geometry matrix and temperature measurements

References

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[2] Christian Raya, „Modelisation et Optimization de Transistor Bipolaires a Heterjonction Si/SiGeC Ultra Rapides pour Applications Millimetriques," PhD Thesis, L' Universite Bordeaux I. 4. juillet 2008

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[4] N. F. Mott, „Note on the Contact between a Metal and an Insulator or Semiconductor," Proc. Cambr. Philos. Soc. 34, 568, (1938)

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[6] Vladimir Milovanovic and Ramses van der Toorn, „On p-n Junction Depletion Capacitance Parameter Extraction Strategies," Proc. 27th Int. Conf. on Microelectronics (MIEL), Nis, Serbia, 16-19 May 2010, pp. 87-90, see also: Solid State Electronics, June 2010, Vol. 54, Issue 6, pages 665-670

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