Calculation of standard deviations and correlation coefficients for bipolar transistors

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Agenda

- Introduction
- Simulation test bench
- Definitions, statistics and theory
- Flow for statistical parameter calculation
- Results of calculations for two NPN transistors
- Conclusions and next steps

Introduction

For Monte Carlo Simulation statistical model parameters are needed.

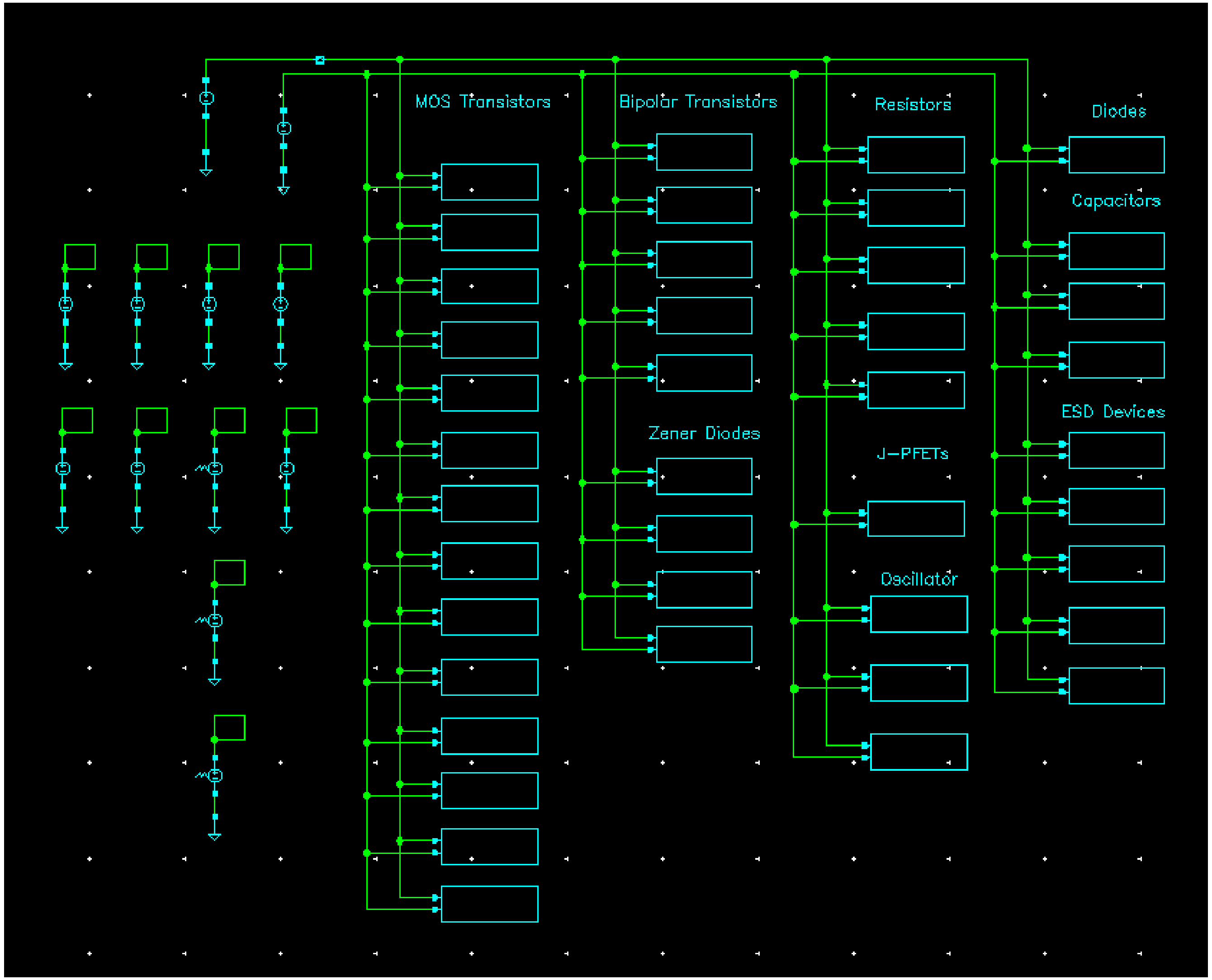
- local parameter variations for device mismatch
- global parameter variations for process variations
- correlation parameters for the dependency between global parameters

For statistical methods like yield prediction etc. the accuracy of all statistical parameters is crucial.

Introduction

- until now, it is hard to calculate standard deviations and correlation coefficients for global model parameter variations that are based on statistical measurements
- from PCM measurements the **standard deviations** and **correlation coefficients** of **PCM parameters P** are well known
- for Monte Carlo simulation the standard deviations and correlation coefficients of **model parameters M** are needed
- idea: Set up a **linear equation** that describes the relations between the standard deviations and correlation coefficients of M and P and solve it (also known as “backward propagation of variances” method)

Simulation Test Bench in Cadence Environment

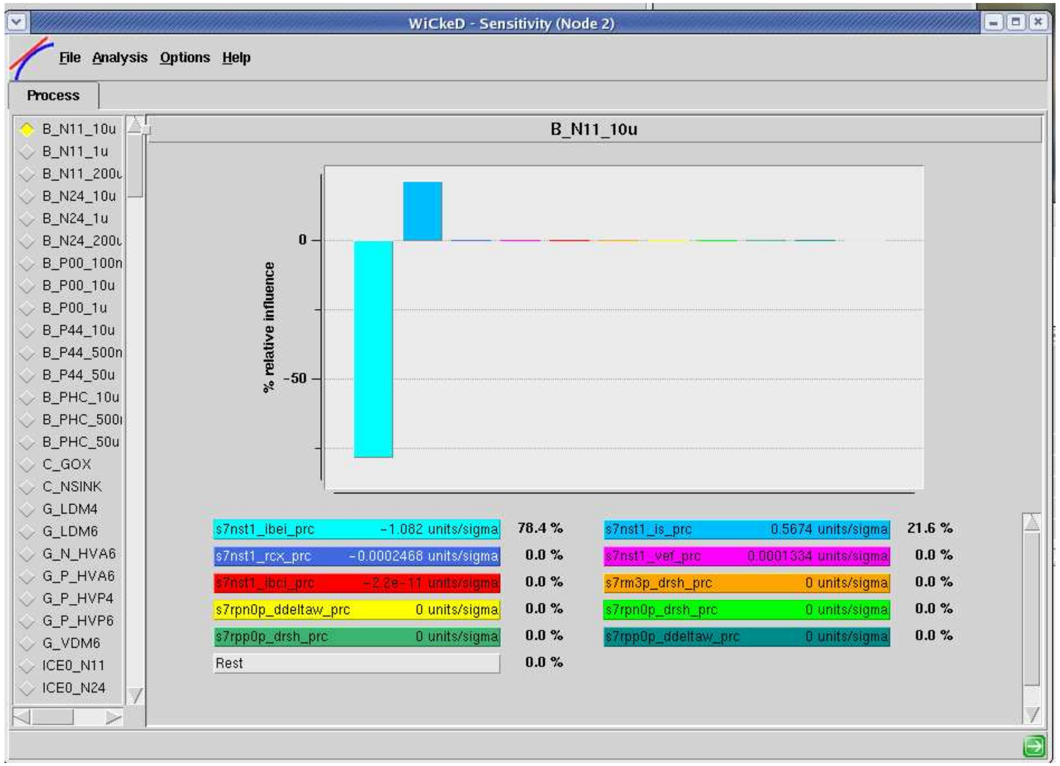


Output:
PCM parameters

BETA_N11
IC_N11
VEA_N11
VCESAT_N11
BETA_N24
IC_N24
VEA_N24
VCESAT_N24
…

Definitions, Statistics and Theory; Sensitivity Matrix

In general a simulated PCM parameter depend on several model parameters



dependencies of PCM parameters on model parameters in the **sensitivity matrix S**

$$S = \begin{bmatrix} \frac{\partial P_1}{\partial M_1} & \cdots & \frac{\partial P_1}{\partial M_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial M_1} & \cdots & \frac{\partial P_n}{\partial M_m} \end{bmatrix}$$

Definitions, Statistics and Theory

$$S \cdot M = P$$

linear approach for relation between model parameter and PCM parameter variations

, with

$$M = (\vec{m}_1, \vec{m}_2, \ldots, \vec{m}_n)$$

random vector of model parameters (n=number of Monte Carlo runs)

$$P = (\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_m)$$

random vector of PCM parameters (m=number of measurements)

$$Cov(S \cdot M) = S \cdot Cov(M) \cdot S^T = Cov(P)$$

covariance matrix equation

Definitions, Statistics and Theory; Covariance Matrix Equation

from simulations — from PCM data

$$S \cdot \underbrace{Cov(M)}_{} \cdot S^T = Cov(P)$$

to be calculated

$$\Rightarrow \sigma_i^M = \sqrt{Cov(\vec{M}_i, \vec{M}_i)}$$

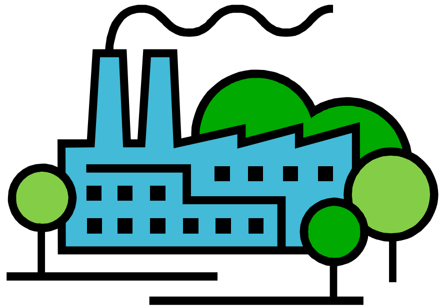
standard deviations

$$\Rightarrow Corr(M) = \left(\frac{Cov(\vec{M}_i, \vec{M}_j)}{\sigma_i^M \sigma_j^M} \right)_{i,j=1,\ldots,n}$$

correlation coefficients

Targets for PCM standard deviations

Simulation



Fab

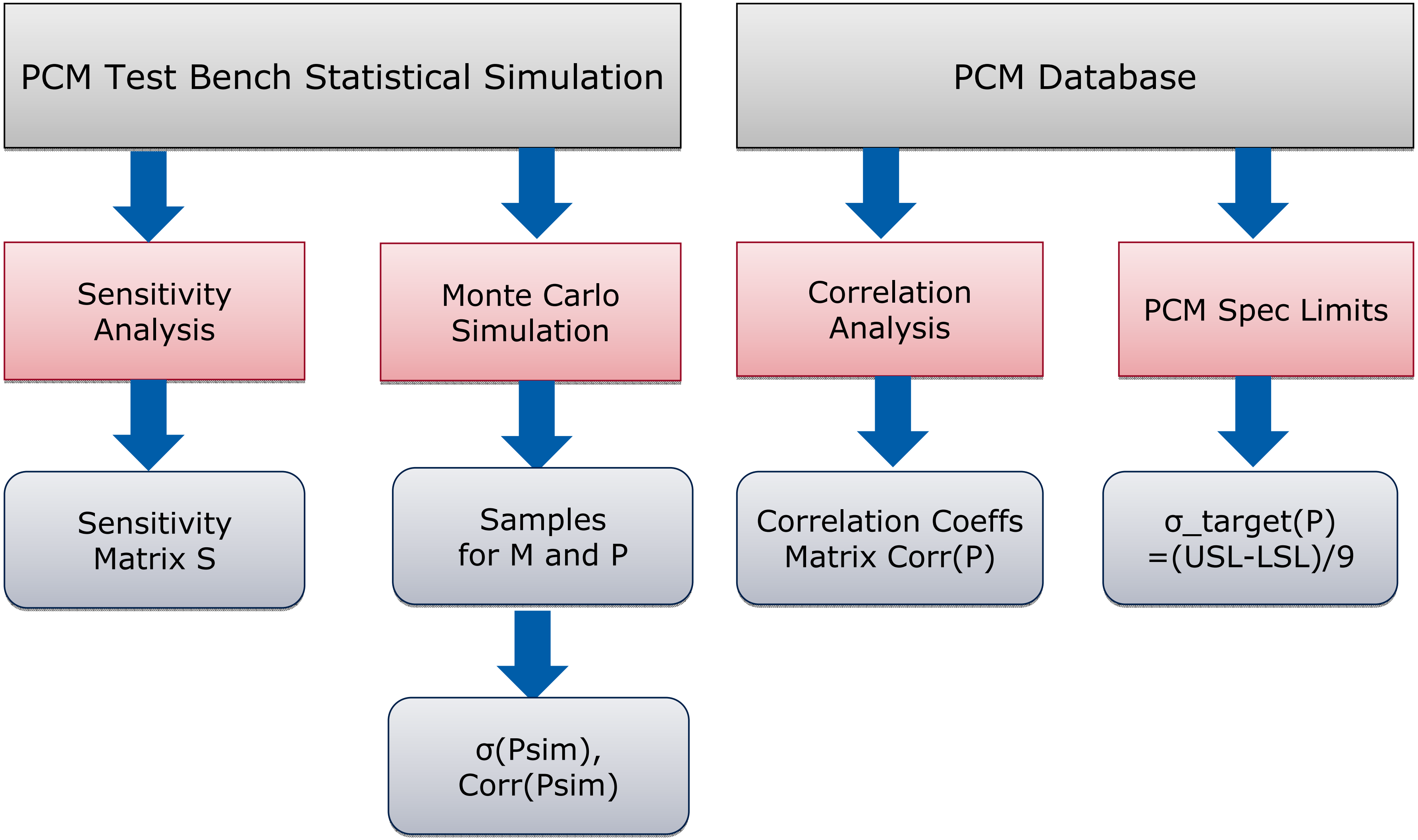
σ= (USL-LSL) / 9
corresponds to
CPK=1.5

Guarantees
CPK~>1.5

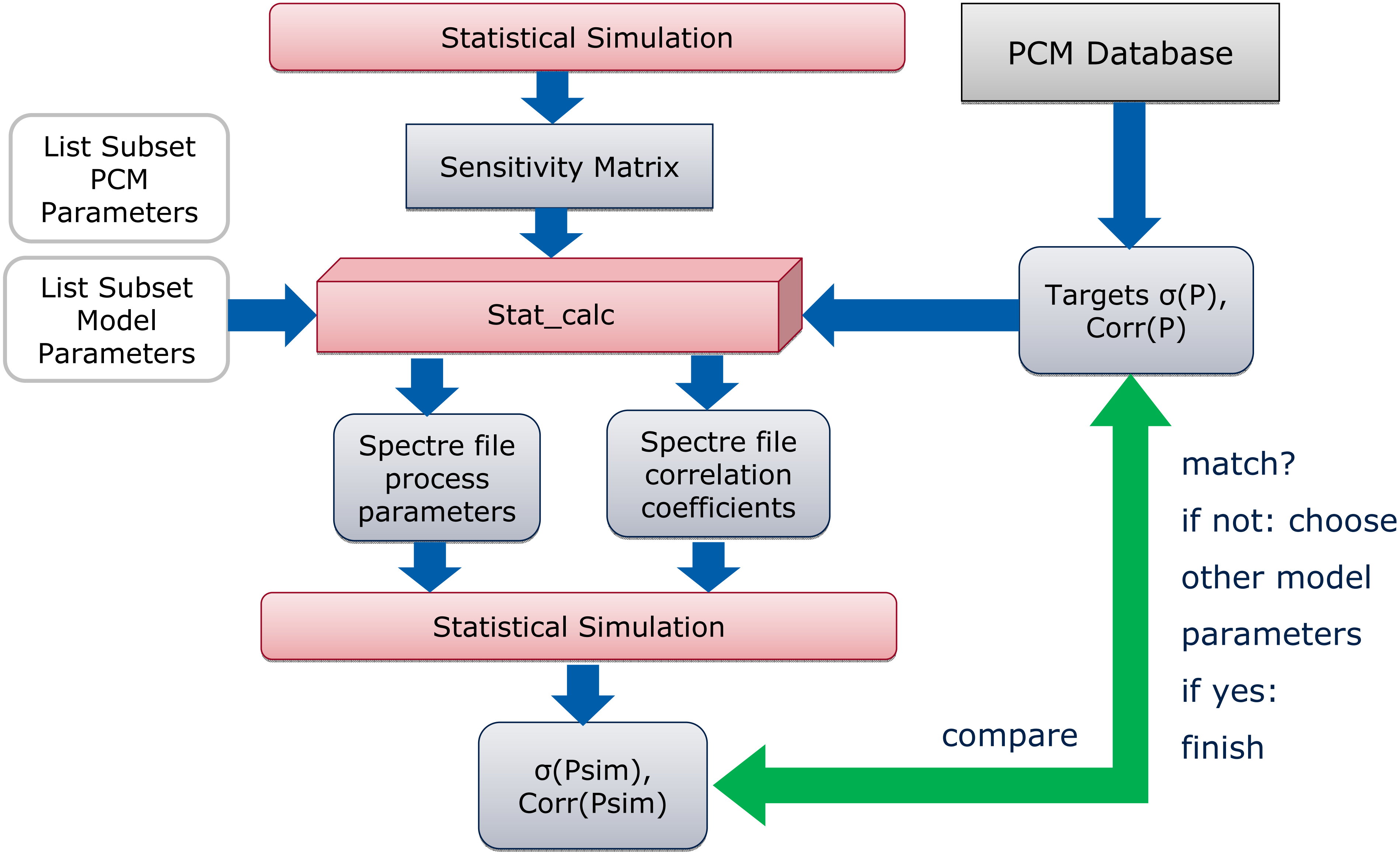
with $$CPK_{PCM} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right)$$ (Process Capability Index)

USL	: Upper Spec Limit	LSL	: Lower Spec Limit
μ	: Mean Value	σ	: Standard Deviation

Flow Statistical Parameter Calculation



Flow Statistical Parameter Calculation

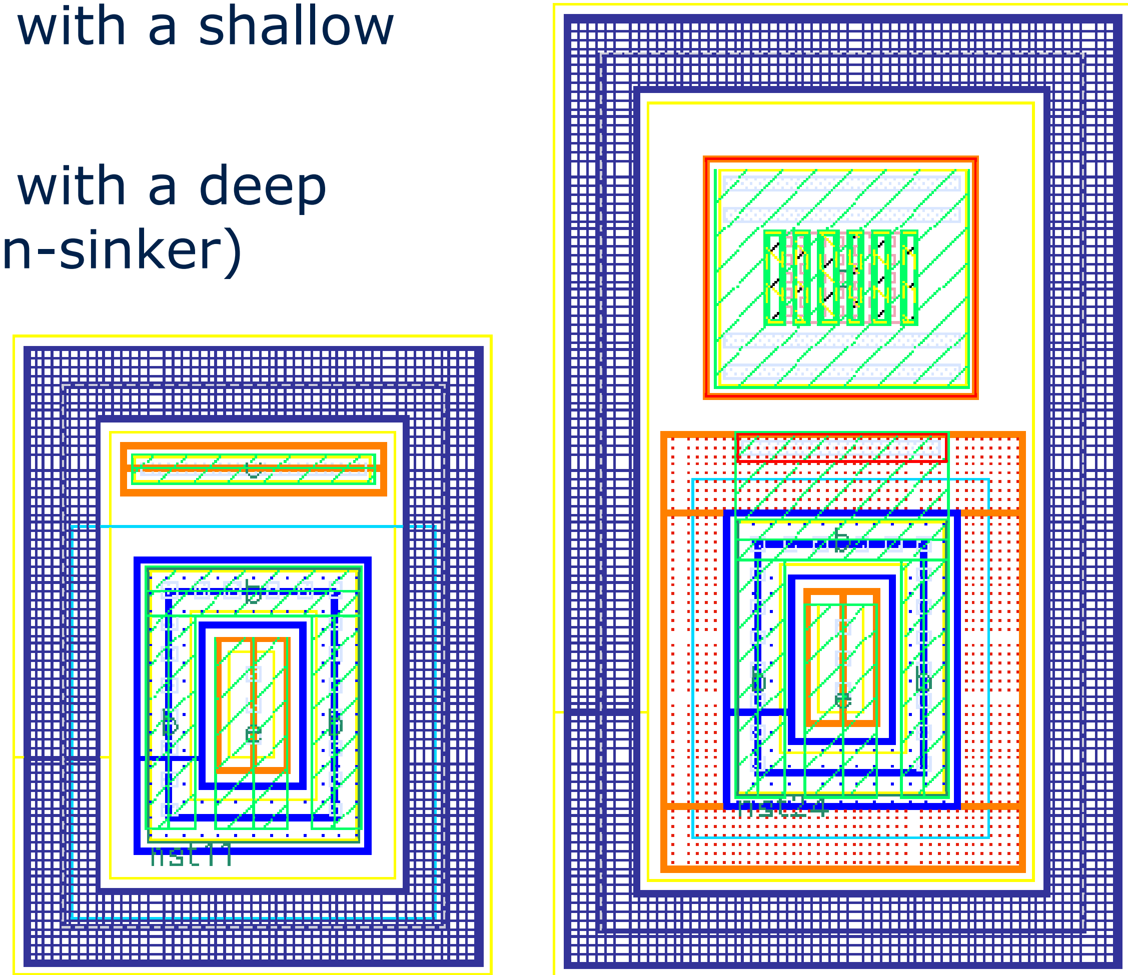


Devices under investigation

2 NPN transistors

- N11: vertical NPN with a shallow collector contact
- N24: vertical NPN with a deep collector contact (n-sinker)

N24



N11

Devices under investigation

PCM parameters

- BETA — Current gain at 1uA collector current
- IC — Collector current at VCE=6V, VBE=0.7V
- VEA — Early voltage
- VCESAT — Collector emitter voltage in saturation

Sensitive model parameters of VBIC bipolar model

- is — Transport saturation current
- ibei — Ideal B-E saturation current
- ibci — Ideal B-C saturation current
- vef — Forward Early voltage

Results: Sensitivity Matrix

	is_N11	ibei_N11	ibci_N11	vef_N11	is_N24	ibei_N24	ibci_N24	vef_N24
BETA_N11	0,55	-0,89	0,00	0,00	0,00	0,00	0,00	0,00
IC_N11	0,89	0,00	0,00	-0,14	0,00	0,00	0,00	0,00
VEA_N11	0,07	-0,13	0,00	1,05	0,00	0,00	0,00	0,00
VCESAT_N11	-0,42	0,08	0,92	-0,02	0,00	0,00	0,00	0,00
BETA_N24	0,00	0,00	0,00	0,00	0,59	-0,88	0,00	0,00
IC_N24	0,00	0,00	0,00	0,00	0,88	0,00	0,00	-0,14
VEA_N24	0,00	0,00	0,00	0,00	0,08	-0,14	0,00	1,03
VCESAT_N24	0,00	0,00	0,00	0,00	-0,57	0,08	0,78	-0,03

N11

N24

Results: Calculated correlation coefficients for model parameters

	is_N11	ibei_N11	ibci_N11	vef_N11	is_N24	ibei_N24	ibci_N24	vef_N24
is_N11	1,00	0,07	0,06	-0,83	0,98	0,13	-0,01	-0,78
ibei_N11	0,07	1,00	0,70	0,08	0,09	0,87	0,81	0,07
ibci_N11	0,06	0,70	1,00	0,00	0,10	0,73	0,74	0,02
vef_N11	-0,83	0,08	0,00	1,00	-0,84	-0,03	0,06	0,94
is_N24	0,98	0,09	0,10	-0,84	1,00	0,16	0,02	-0,81
ibei_N24	0,13	0,87	0,73	-0,03	0,16	1,00	0,86	0,04
ibci_N24	-0,01	0,81	0,74	0,06	0,02	0,86	1,00	0,10
vef_N24	-0,78	0,07	0,02	0,94	-0,81	0,04	0,10	1,00

(circle) candidates for reduction of parameters; one variation factor for both devices

Results: Simulated PCM Correlation Matrix

	BETA_N11	IC_N11	VEA_N11	VCESAT_N11	BETA_N24	IC_N24	VEA_N24	VCESAT_N24
BETA_N11	1,00	0,52	-0,44	-0,77	0,91	0,49	-0,41	-0,87
IC_N11	0,52	1,00	-0,86	-0,39	0,51	0,98	-0,83	-0,57
VEA_N11	-0,44	-0,86	1,00	0,27	-0,39	-0,87	0,95	0,44
VCESAT_N11	-0,77	-0,39	0,27	1,00	-0,78	-0,36	0,27	0,77
BETA_N24	0,91	0,51	-0,39	-0,78	1,00	0,50	-0,40	-0,91
IC_N24	0,49	0,98	-0,87	-0,36	0,50	1,00	-0,86	-0,57
VEA_N24	-0,41	-0,83	0,95	0,27	-0,40	-0,86	1,00	0,43
VCESAT_N24	-0,87	-0,57	0,44	0,77	-0,91	-0,57	0,43	1,00

Results: Differences Simulated-Measured Correlation Matrix

	BETA_N11	IC_N11	VEA_N11	VCESAT_N11	BETA_N24	IC_N24	VEA_N24	VCESAT_N24
BETA_N11	0,00	0,02	-0,03	0,00	0,01	0,02	-0,03	0,00
IC_N11	0,02	0,00	0,00	-0,04	0,04	0,00	-0,02	-0,03
VEA_N11	-0,03	0,00	0,00	0,03	-0,02	0,00	0,01	0,03
VCESAT_N11	0,00	-0,04	0,03	0,00	0,00	-0,05	0,04	-0,01
BETA_N24	0,01	0,04	-0,02	0,00	0,00	0,04	-0,01	0,00
IC_N24	0,02	0,00	0,00	-0,05	0,04	0,00	-0,01	-0,04
VEA_N24	-0,03	-0,02	0,01	0,04	-0,01	-0,01	0,00	0,02
VCESAT_N24	0,00	-0,03	0,03	-0,01	0,00	-0,04	0,02	0,00

Results: simulated standard deviations

Model Parameter Variations

Parameter Name	Sigma	Unit
is_N11	4,439	%
ibei_N11	10,029	%
ibci_N11	11,755	%
vef_N11	1,182	V
is_N24	4,993	%
ibei_N24	10,516	%
ibci_N24	8,310	%
vef_N24	1,220	V

PCM Parameter Deviations

Parameter Name	Sigma Target	Sigma Simulated	Unit	rel. Deviation
BETA_N11	8,889E+00	9,222E+00		3,75%
IC_N11	1,111E-06	1,123E-06	A	1,11%
VEA_N11	2,778E+00	2,727E+00	V	-1,82%
VCESAT_N11	1,111E-03	1,181E-03	V	6,29%
BETA_N24	9,444E+00	9,666E+00		2,34%
IC_N24	1,333E-06	1,329E-06	A	-0,36%
VEA_N24	2,778E+00	2,698E+00	V	-2,85%
VCESAT_N24	1,000E-03	1,014E-03	V	1,41%

Conclusions and next steps

- conclusions
  - a new method was applied to calculate standard deviations and correlation coefficients in one step
  - the method is automatable, update of model parameter can be done very fast
  - final simulation shows very good agreement with target values

- next steps
  - use DOE PCM data for better target correlations
  - investigate the influence of tester offsets
  - apply the new method to other technologies
  - create a GUI for the program

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