Coupled extraction of RE and RTH
based on DC output curves

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Bipolar Arbeitskreis 2013, Frankfurt (Oder)
Outline

• Introduction
• Method
• Results
• Summary
Introduction

• Several methods for $R_E$ and $R_{TH}$ extraction exist
  • Most for $R_E$ based on S-parameters

• Also combined methods exist

• Here, a new method based on a simple self-heating model is presented
  • Only based on DC output curves

• Basic concept
  • Calculation of temperature increase from $I_B(V_{BE})$ values
  • Calculation of $R_E$ from dissipated power and difference of $V_{CEi}$ and $V_{CE}$
Method

Base current

• In forward active mode $I_B = f(V_{BEi}, T, V_{BCi})$

• $V_{BEi} = V_{BE} - I_E^* R_E - I_B^* R_B$

• Known parameters: $I_{BES}(T_0), m_{BE}, \zeta_{BET}, V_{gB} \rightarrow I_{BES}(T)$

• Measured values: $V_{BE}, I_B, I_E(= I_C)$

Self-heating

• Simplified model: $\Delta T = I_C^* V_{CEi}^* R_{TH}$

• $V_{CEi} = V_{CE} - I_E^* R_E - I_C^* R_{Cx}$

• Measured values: $V_{CE}, I_E(= I_C)$
• Measurement at fixed $I_C$ (using a control script adjusting $V_{BE}$)
  • or forced $I_B$, if temperature dependence of beta is low

• Assuming $I_B$ only consists of $I_{BE}$, $\Delta T$ can be calculated with known $R_E$ from measured $I_B(V_{BE})$

\[
I_B = I_{BEs}(T) \exp\left(\frac{V_{BE} - I_E R_E}{m_{BE} V_T}\right) \quad \text{(solved for $T$ with Newton-method)}
\]

• Since fixed $I_C$ (assuming $R_{TH}(T) = \text{const}$ and $R_E(T) = \text{const}$)

\[
\frac{\Delta T}{I_C} = V_{CEi} R_{TH} = (V_{CE} - I_E R_E) R_{TH} = V_{CE} R_{TH} - I_E R_E R_{TH}
\]

\[
\Rightarrow \Delta T \sim V_{CEi} \quad \text{and} \quad \Delta T = f_{lin}(V_{CE})
\]

• Since generally $R_{TH}(T) \neq \text{const}$ and $R_E(T) \neq \text{const} \Rightarrow$ no ideally linear function $\Rightarrow$ see results
• Linear extrapolation of $\Delta T$ to 0 $\Rightarrow$ $\Delta T(V_{CE0}) = 0$

• In high current region, $\Delta T$ is wrong due to additional base current components
• BUT: $R_E$ is not known

• $R_E$ can be calculated from $V_{CE0}$

$$R_E = \frac{V_{CE0}}{I_E}$$

• $\Delta T$ and $V_{CE0}$ are calculated by using an initial value of $R_E$

• Defining $R_{E, temp}$ for calculating $\Delta T$ and $R_{E, extr}$ from $V_{CE0}$

$$R_{E, temp} \Rightarrow \Delta T_{extr} \Rightarrow V_{CE0} \Rightarrow R_{E, extr}$$

$$I_B = I_{BEs}(T_{extr}) \exp\left(\frac{V_{BE} - I_E R_{E, temp}}{m_{BE} V_T, extr}\right), \quad R_{E, extr} = \frac{V_{CE0}}{I_E}$$

=> for one $R_{E, temp} \Rightarrow R_{E, temp} = R_{E, extr} = R_E$
• Automated optimization for $R_E$
  • Using an iteration
  • Convergence criteria change of $R_E$ less than a specified value

• Using Newton-like iteration
• Using some kind of bisection method

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Assumed $R_E$</th>
<th>Calculated $R_E$</th>
<th>Iteration</th>
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<td>10</td>
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</table>

Final value after $R_E$ iteration

Starting point $R_{E, temp}=0$
Influence of $R_{Cx}$

- $V_{CEi} = V_{CE} - I_E R_E - I_C R_{Cx}$

- Two methods:
  - Using known $R_{Cx}$ from test structures
  - $R_{Cx}$ including in extraction

![Graph showing weakly dependent and dependent behavior of $R_{E}$ and $R_{th}$ with $R_{Cx}$]
• Using two different forced $I_B$ or two constant $I_C$ => $I_{C1}$ and $I_{C2}$

• Same $P_{diss}$ (=> same $\Delta T$) for both
  • Choosing a fixed $\Delta T$ and $I_{C1} \neq I_{C2}$ => $V_{CE1} \neq V_{CE2}$
  • But same $R_E$, $R_{Cx}$ and $R_{TH}$ (as function of $T$)
• Calculation of $R_{Cx}$ from temperature increases

\[ P_{diss1} = P_{diss2} \]

\[ I_C1(V_{CE1} - R{EIF1} - R_{Cx}Ic1)R_{TH} = I_C2(V_{CE2} - R{EIF2} - R_{Cx}Ic2)R_{TH} \]

\[ R_{Cx} = \frac{I_C1V_{CE1} - I_C2V_{CE2} - R{EIF1} - I_C1I{EIF1} - I_C2I{EIF2}}{I^2_C1 - I^2_C2} \]

• Numerical optimization generally fails for experimental results

• Influence of $R_{Ci}$
  • Voltage drop for $V_{BC} <$ punch-through
  • To be investigated
Results

• Comparison versus model

• Test cases:
  • Ideal case: \( R_E(T) = \text{const}, \ R_{TH}(T) = \text{const}, \ R_C = R_B = 0 \)
  • External resistances: \( R_E(T) = \text{const}, \ R_{TH}(T) = \text{const}, \ R_C \neq 0, \ R_B \neq 0 \)
  • T-dep RE: \( R_E(T) \neq \text{const}, \ R_{TH}(T) = \text{const}, \ R_C \neq 0, \ R_B \neq 0 \)
  • Realistic case: \( R_E(T) \neq \text{const}, \ R_{TH}(T) \neq \text{const}, \ R_C \neq 0, \ R_B \neq 0 \)

• \( R_C \) is assumed to be known in all cases

• Always four different values for \( I_B \)
  • Mean value of \( R_E \) and \( R_{TH} \) from each IB used
Ideal case

External resistances
Temperature dependent $R_E$

Realistic case
Comparison of results

<table>
<thead>
<tr>
<th>Test case</th>
<th>$R_E$ (model)</th>
<th>$R_E$ (extract)</th>
<th>$\zeta_{RE}$ (model)</th>
<th>$\zeta_{RE}$ (extract)</th>
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</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>3.023 Ω</td>
<td>2.97 Ω</td>
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<tr>
<td>Ext. res</td>
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<tr>
<td>Realistic</td>
<td>3.023 Ω</td>
<td>3.0 Ω</td>
<td>-0.96</td>
<td>-0.96</td>
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</table>

<table>
<thead>
<tr>
<th>Test case</th>
<th>$R_{TH}$ (model)</th>
<th>$R_{TH}$ (extract)</th>
<th>$\zeta_{RTH}$ (model)</th>
<th>$\zeta_{RTH}$ (extract)</th>
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<tbody>
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<tr>
<td>Ext. res</td>
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<tr>
<td>Realistic</td>
<td>1.98 K/mW</td>
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<td>0.5</td>
<td>0.47</td>
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</tbody>
</table>
Measurements

- Meaningful scaling results
Summary

• Pros
  • Simple measurements, DC forced $I_B$
  • Robust => without $R_C$ in the iteration always converges
  • Based on model equations => final model will agree well

• Cons
  • Temperature dependence of $R_E$ and $R_{TH}$ affects linear fit of $\Delta T(V_{CE})$
  • $R_{Cx}$ should be known in advance
  • Based on model equations => physical value of extracted parameters dependent on model equations for self-heating
  • Limited range during extraction
    • $V_{BE}$ large enough for self-heating but below high-current effects
    • $V_{CE}$ between saturation and breakdown => linear fit

• To do: evaluation for large geometry and technology range
Acknowledgements

• European Commission within the FP7-IP DOTSEVEN (ICT-316755)

• German Research Foundation (DFG) in the Collaborative Research Center 912 "Highly Adaptive Energy-Efficient Computing"

• German Ministry of Research and Education (BMBF) within the CoolSilicon Excellence Cluster