

Basic Principle of Model Parameter Extraction - Application to the Knee Current of SGP Model with QucsStudio

32nd AKB Workshop
Crolles - November 14/15, 2019

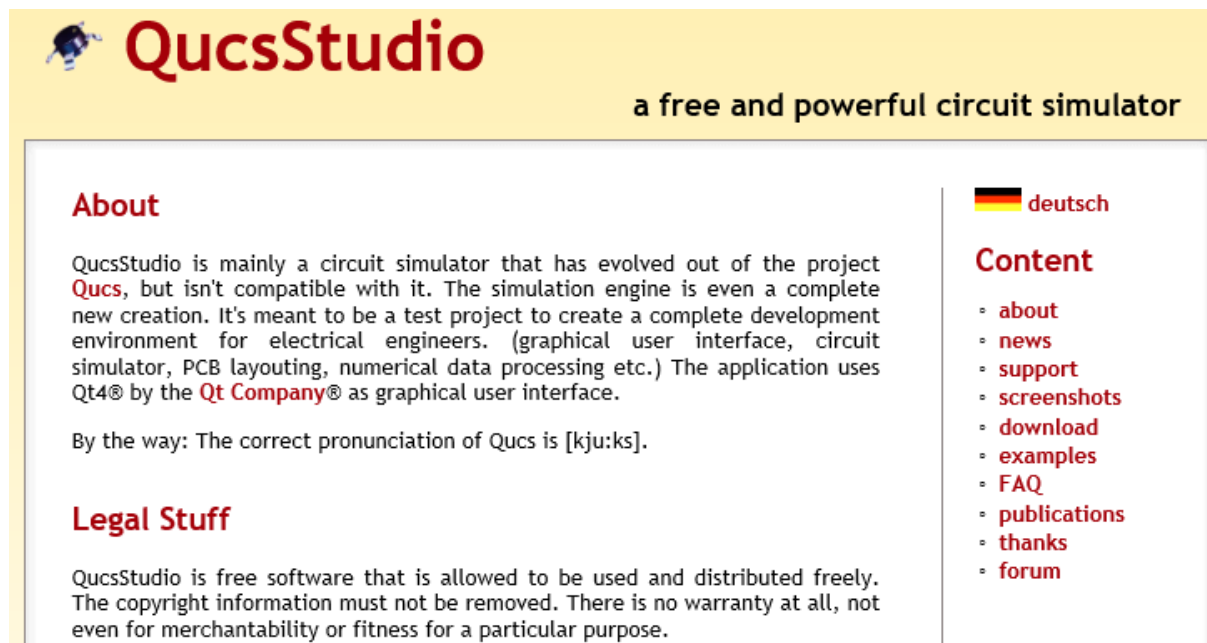
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ST Confidential

- Reminder on the basic principles for the extraction of model parameters
- Application to the extraction of the forward knee current I_{KF} of the SPICE Gummel-Poon (SGP) model
- In complement to [1], [2] and [3] the Free and Open Source Software (FOSS) QucsStudio [4], [5] is used to implement and validate the extraction procedure



The screenshot shows the QucsStudio website. At the top left is the QucsStudio logo, a small robot icon. The main heading is "QucsStudio" in a large, bold, red font, followed by the tagline "a free and powerful circuit simulator" in a smaller black font. Below this, there are two columns of text. The left column is titled "About" and contains a paragraph describing the simulator's development and its use of Qt4. Below this is a note about the correct pronunciation of "Qucs". The right column is titled "Content" and lists various links: about, news, support, screenshots, download, examples, FAQ, publications, thanks, and forum. A small German flag icon is next to the word "deutsch".



The screenshot shows the QucsStudio Forum website. The header is "QucsStudio Forum" in white text on a dark blue background. Below the header, there is a "Forum" section with a light blue background. The main content area is titled "QucsStudio Forum" and contains the text "About a free and powerfull Circuit Simulator". Below this, there is a "Forum" section with the text "Discussions about QucsStudio".

■ Objectives

- Independently of the model used, we want **reliable** model parameters
- **Reliable** meaning both **physical** and **accurate** model parameters
- *Do not forget that a physics-based model with inaccurate model parameters can be worse than a less accurate compact model but with physical model parameters*

■ Constraints

- All models have their own limitations
- Measurements are more or less accurate
- **Therefore, how to determine model parameters both accurate and physics-based taking into account the limits of the compact models and the inaccuracy of measurements?**

■ Key solution

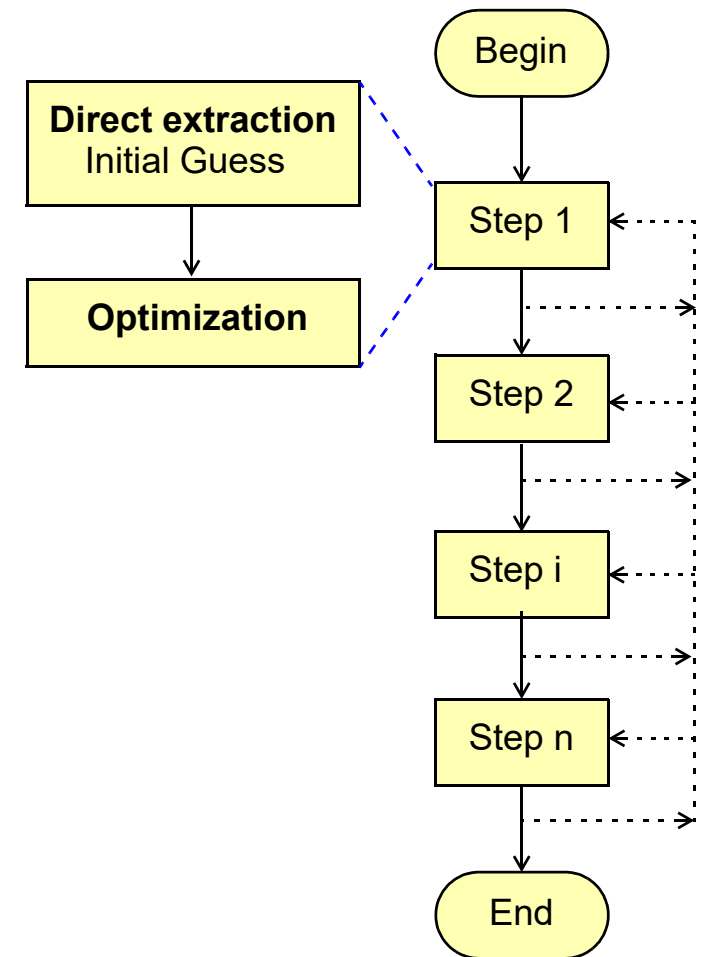
- Developing direct extraction procedures using e.g. linear regression (Appendix A) gives the solution without any iteration loop, without initial guesses and then avoids correlation between model parameters...

■ Advantages

- **Easy parameter extraction**, the only difficulty being to find the adequate transformations for linearizing the equations of the compact models, an important job of modeling engineers.
 - **Allows to validate both compact models and measurements.**
 - If the theory predicts that a given characteristic must be linear and if the measurements are also linear, that validates both the measured data and the model equations.
 - If it was not the case, that allows to alert the modeling engineers: either it is a model limitation or a measurements issue (limitation of the equipments, wrong test structures or measurement setup), or both.
- ⇒ **In this case accurate extraction of model parameters will be not possible.**

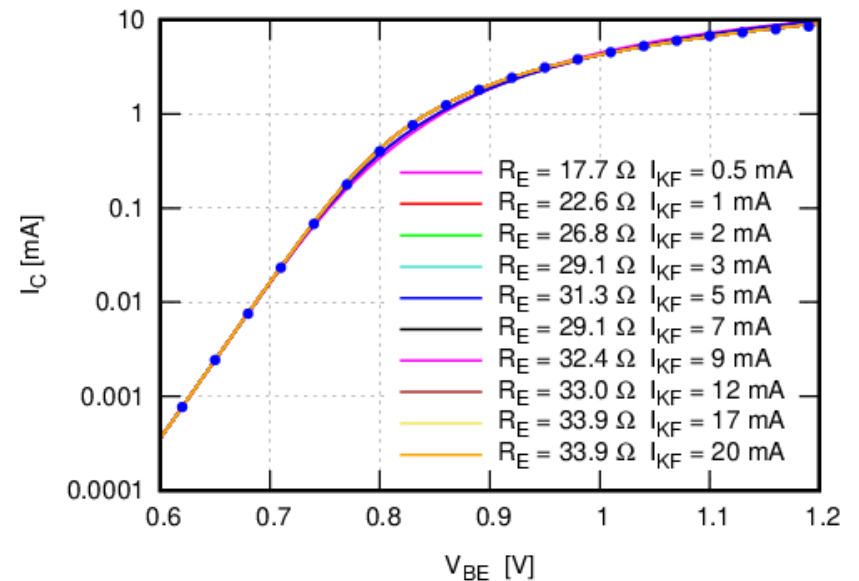
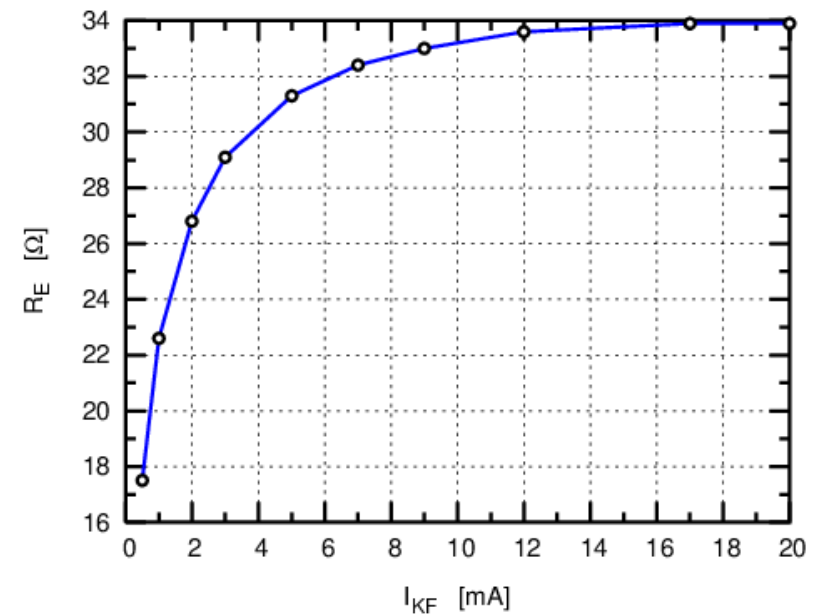
Basic principle of parameter extraction (2/2)


- The parameter extraction is performed in several steps
 - With possible loops between the different steps
- At each extraction step
 - a given set of model parameters is determined
 - from electrical characteristics (DC, AC, noise, temperature) where the set of extracted parameters have the most impact.
- Each step is divided in 2 parts
 - The first part consists of a direct extraction of the model parameters (initial guess).
 - The second part uses non-linear least-squares algorithms for the determination of the parameters with initial guess coming from the first part.



Application to the knee current I_{KF} of the SGP model

- Why to choose I_{KF} as example?
- Because it is a typical case where global optimization could give unrealistic I_{KF} values depending on the values of the emitter resistance R_E .
- From measurement, by optimizing the collector current I_C at high-current, several (I_{KF}, R_E) combinations give similar *fit*.



- Why  current and for what?

- In SGP model, the forward knee current I_{KF} is used to model the high-injection effects
 - High-injection effects occur when injected minority carriers are greater than the doping level.

■ Model formulation

- Forward mode ($V_{BEi} > 0$ and $V_{BCi} = 0$ V)
- No Early effect $V_{AF} = V_{AR} = \infty$
- The collector current can be written

$$I_C = \frac{I_S}{q_b} \cdot \left(e^{\frac{V_{BEi}}{V_T}} - 1 \right) \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{q_1}{2} \cdot \{1 + \sqrt{1 + 4q_2}\}} \quad \text{with} \quad (1)$$

$$q_1 = \frac{1}{1 - \frac{V_{BEi}}{V_{AR}} - \frac{V_{BCi}}{V_{AF}}} \approx 1 \quad \text{Early effects} \quad (2)$$

$$q_2 = \frac{I_S}{I_{KF}} \cdot \left(e^{\frac{V_{BEi}}{V_T}} + 1 \right) + \frac{I_S}{I_{KR}} \cdot \left(e^{\frac{V_{BCi}}{V_T}} + 1 \right) \approx \frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{V_T}} \quad \text{High-current effects} \quad (3)$$

I_{KF} explained (2/3)

- From (1), (2) and (3) the collector current in forward mode can be written

$$I_C = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} \quad (4)$$

- Asymptotic value at low currents $I_C \ll I_{KF}$

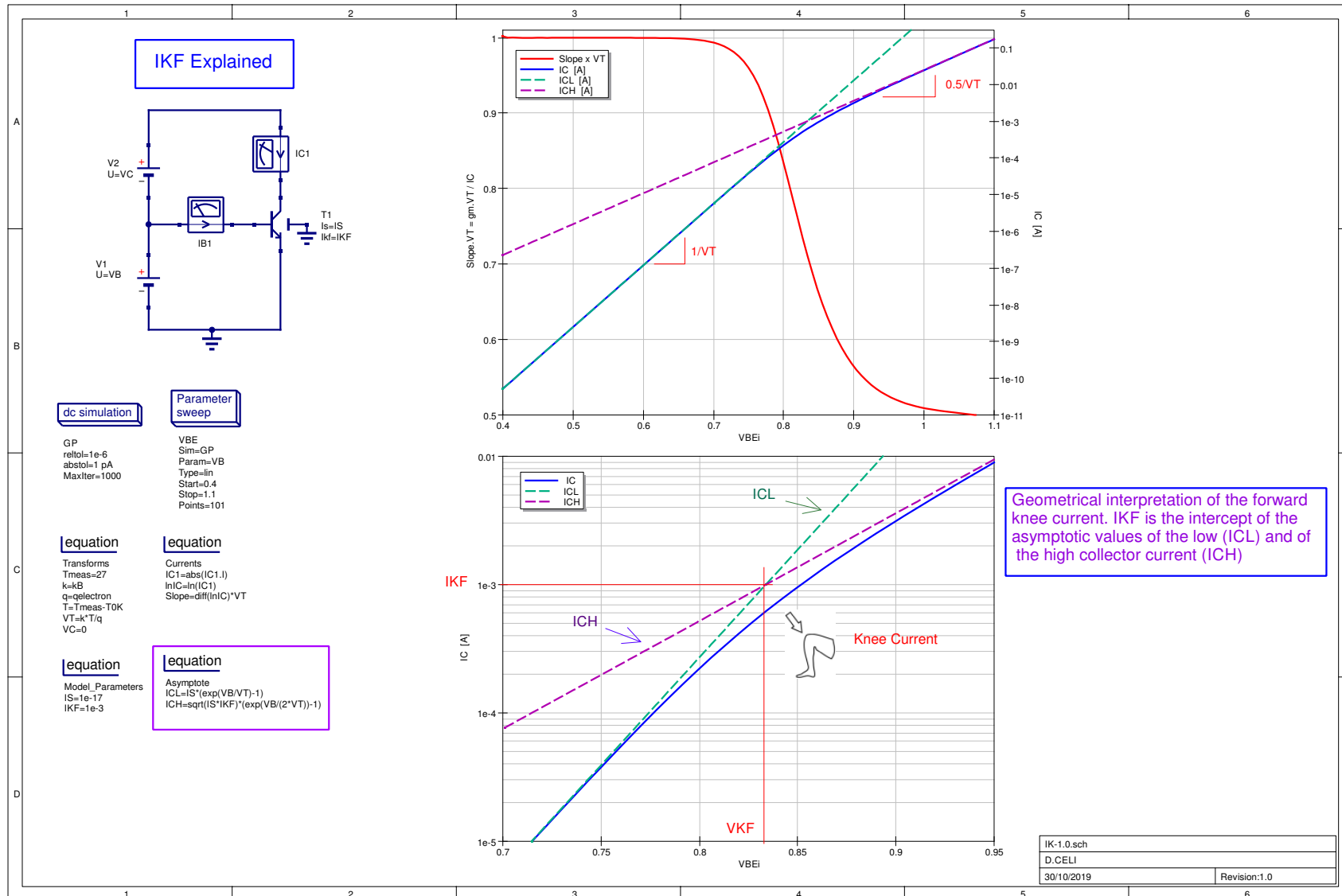
$$I_{CL} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \{1 + \sqrt{1}\}} = I_S \cdot e^{\frac{V_{BEi}}{V_T}} \quad (5)$$

- Asymptotic value at high current $I_C \gg I_{KF}$

$$I_{CH} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ \sqrt{4 \cdot \frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{2 \cdot \sqrt{\frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{2 \cdot V_T}}}} = \sqrt{I_S \cdot I_{KF}} \cdot e^{\frac{V_{BEi}}{2 \cdot V_T}} = I_{SH} \cdot e^{\frac{V_{BEi}}{2 \cdot V_T}} \quad (6)$$

I_{KF} explained (3/3)

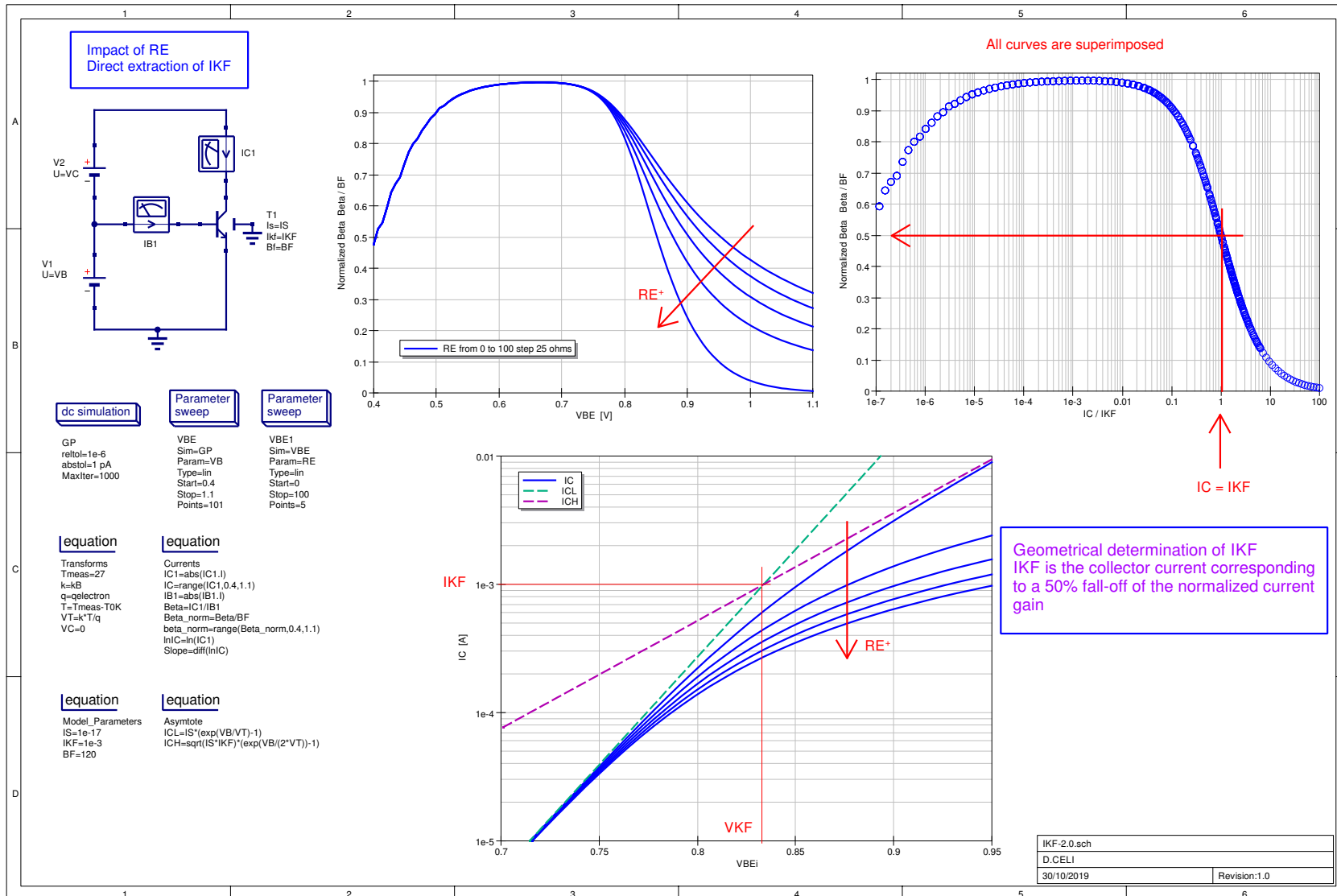
■ QucsStudio worksheet for I_{KF} explanation



- I_{KF} affects the bending of I_C at high V_{BE}
- And therefore the current gain β fall-off at high-current
 - $\beta = I_C/I_B$, I_B not affected by high-injection effects as the doping level of the emitter is too high.
- But unfortunately, as already shown in slide 4, other important parasitic effects also affect the curvature of I_C at high currents
 - Voltage drop in series resistances (R_E , R_B , R_C)
 - Self-heating (SH)
- Now, the main question is how to extract I_{KF} without to be impacted by these parasitic effects?
- For that, we will analyze the dependence of the normalized current gain β/B_F , at $V_{BC}=0V$, versus V_{BE} and I_C
 - Simulations performed with QucsStudio

β/B_F versus V_{BE} and I_C (1/2)

■ QucsStudio worksheet impact of R_E on the forward current gain



β/\mathbf{B}_F versus V_{BE} and I_C (2/2)

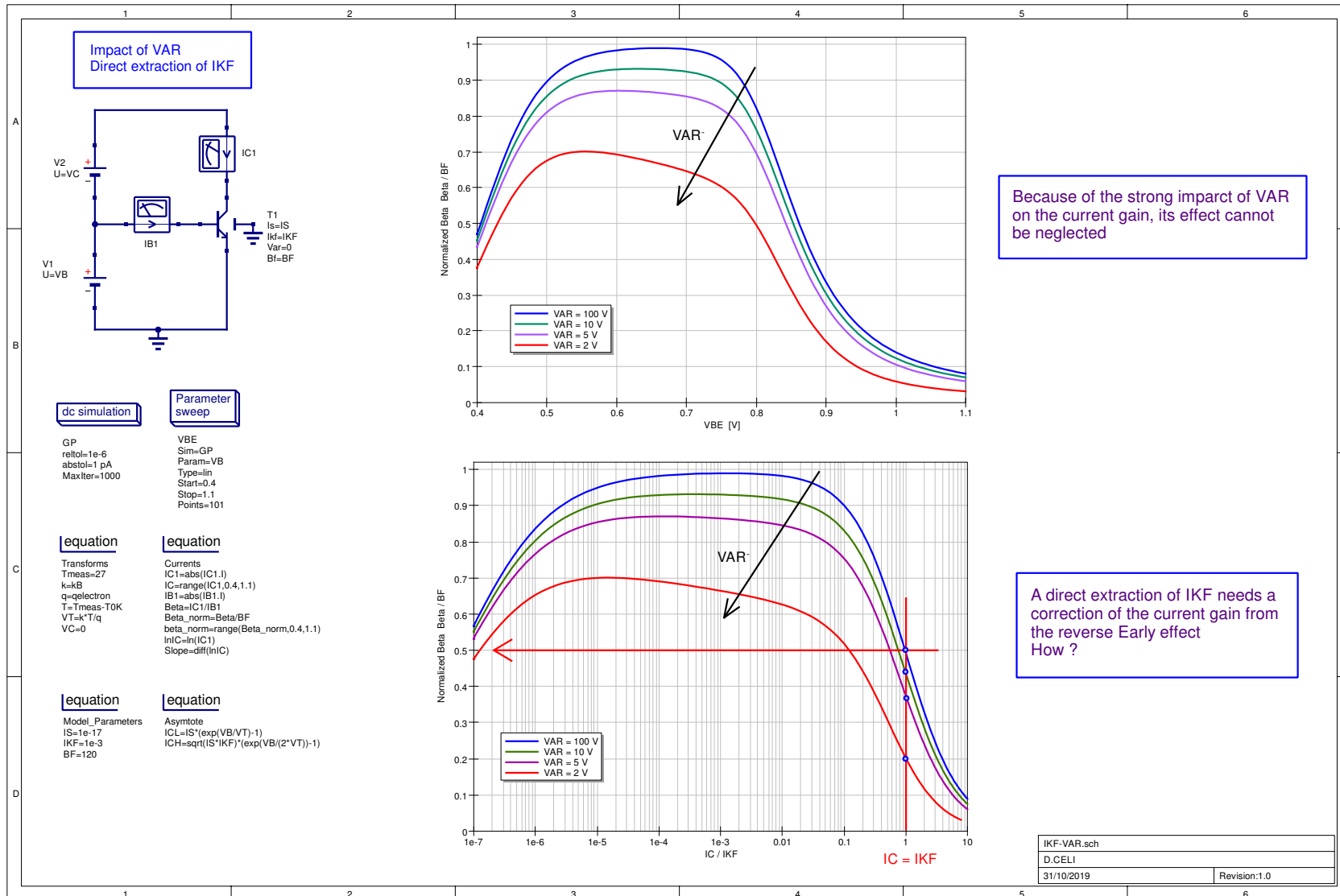
10/28

- Main important results.
- β/\mathbf{B}_F versus V_{BE} is strongly impacted by the emitter resistance, but also by the base resistance (not shown here).
- But in the contrary, β/\mathbf{B}_F versus I_C (@ $V_{BC}=0V$) is not affected by R_E and R_B , and I_{KF} is the value of the collector current corresponding to a 50% fall-off (at high-current) of the normalized current gain.
- This method is a direct method allowing to have a first order value of I_{KF} , without any calculation
- But it is not so simple, up to now the impact of the reverse Early voltage has been neglected ($V_{AR} = \infty$, $q_1 = 1$)

$$q_1 = \frac{1}{1 - \frac{V_{BEi}}{V_{AR}}} \quad (7)$$

- This approximation is not valid for modern BJTs or HBTs

■ QucsStudio worksheep showing the impact of V_{AR} on the forward current gain



How to correct the impact of V_{AR} ?

■ Impact of V_{AR} on I_C

- From (1) and (7) we can write

$$I_C = \frac{I_S}{q_b} \cdot \left(e^{\frac{V_{BEi}}{V_T}} - 1 \right) \approx \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{q_1}{2} \cdot 1 + \sqrt{1 + 4q_2}} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{q_1 \cdot q_{b}^*} = \frac{I_S \cdot \left\{ 1 - \frac{V_{BEi}}{V_{AR}} \right\} \cdot e^{\frac{V_{BEi}}{V_T}}}{q_{b}^*} \quad \text{with} \quad (8)$$

$$q_{b}^* = \frac{1 + \sqrt{1 + 4q_2}}{2} \quad \text{is the normalized majority base charge without Early effect} \quad (9)$$

■ Correction of I_C from the impact of V_{AR}

- From (8) the corrected I_{C^*} is defined by

$$I_{C^*} = \frac{I_C}{1 - \frac{V_{BEi}}{V_{AR}}} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{q_{b}^*} \quad (10)$$

- Therefore, the correction of I_C from the reverse Early voltage needs to know the internal base emitter voltage V_{BEi}

Estimation of V_{BEi}

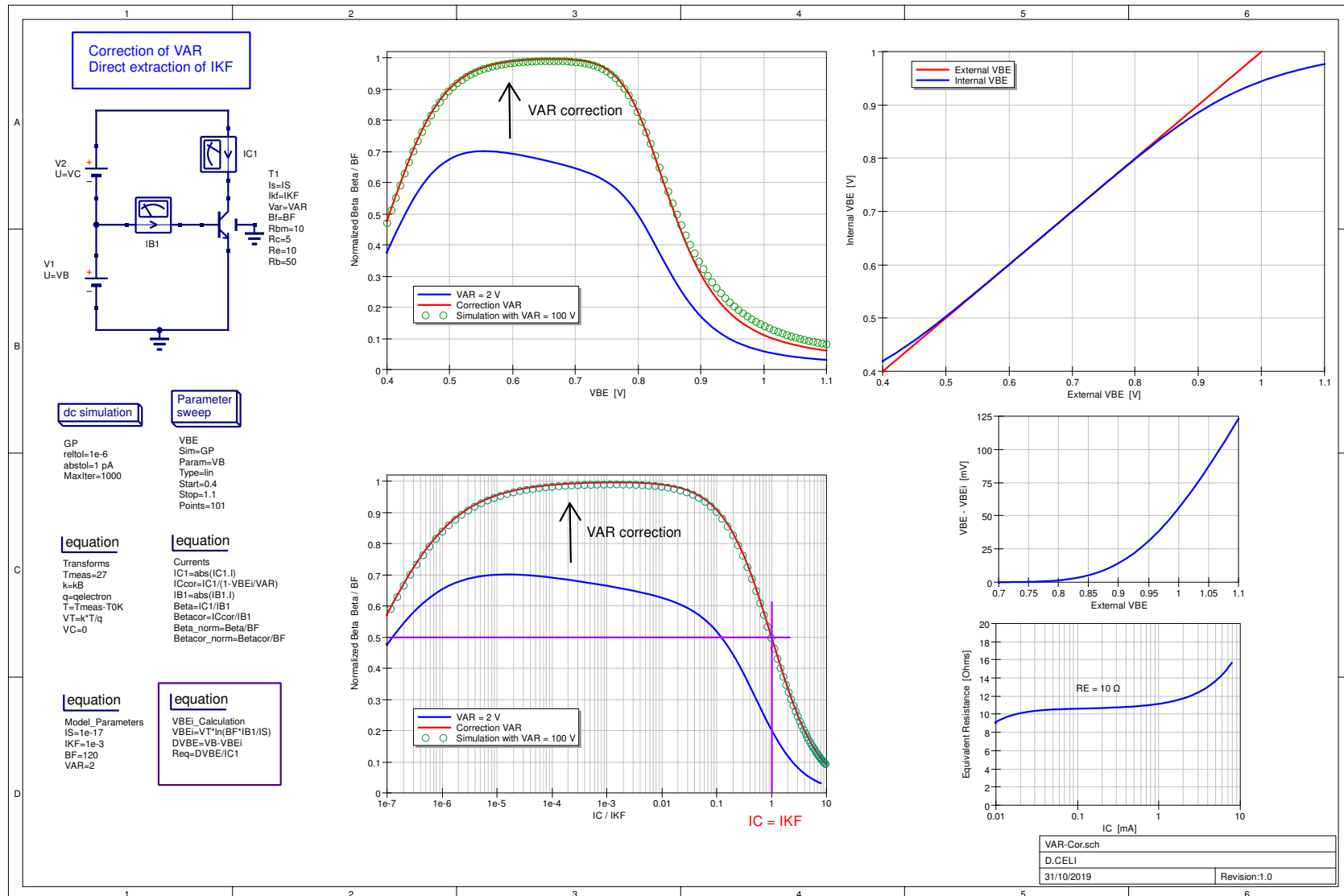
- How to estimate V_{BEi} without to know the access series resistances R_E and R_B ?
- Assumption
 - We assume that at high V_{BE} and $V_{BC} = 0V$ the base recombination current is negligible and that we can write

$$I_B = \frac{I_S}{\beta_F} \cdot e^{\frac{V_{BEi}}{V_T}} \quad (11)$$

- V_{BEi} calculation
 - Knowing I_S and β_F , from (11), it is easy to compute V_{BEi}

$$V_{BEi} = V_T \cdot \ln \left\{ \frac{\beta_F \cdot I_B}{I_S} \right\} \quad (12)$$

■ QucsStudio worksheet correction of V_{AR}



- From all these previous observations, we will be able to define an extraction strategy for I_{KF} without to have to know the values of the access series resistances

■ Assumptions

- SGP model (with its limitations)
- No (or negligible) self-heating
- Sufficient low collector resistance **R_C** to avoid the saturation of the device at V_{BC} = 0V and at high-current

■ Prerequisite model parameters

- Low collector current parameters: **I_S**
- Base current parameters: **B_F**, **I_{SE}**, **N_E**
- Reverse Early voltage **V_{AR}**

■ Comments

- In slide 9 we have observed that the forward current gain, corrected from V_{AR}, $\beta^* = I_{C^*}/I_B$ vs. I_{C*} was independent of the series resistances. I_{C*} is given by (10).
- The idea is not to use β^* , to be not impacted by the possible *non-ideality* of I_B, but the normalized collector current I_{CN*} = I_{C*}/I_{CL*}, where I_{CL*} is simply defined by

$$I_{CL^*} = I_S \cdot e^{\frac{V_{BEi}}{V_T}}$$

(13)

■ Expression of I_{CN*} vs. I_{C*}

- By definition the normalized collector current is given by

$$I_{CN*} = \frac{I_{C*}}{I_{CL*}} \tag{14}$$

- with I_{CL*} given by (13) and

$$\left\{ \begin{aligned} I_{C*} &= \frac{I_C}{1 - \frac{V_{BEi}}{V_{AR}}} = \frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{q_{*b}} \\ q_{*b} &= \frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{I_S}{I_{KF}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\} \end{aligned} \right. \tag{15}$$

- We want a formulation of I_{C*} independent of V_{BEi}, let us write

$$x = I_S \cdot e^{\frac{V_{BEi}}{V_T}} \tag{16}$$

$$(15) \Leftrightarrow \left\{ \begin{aligned} q_{*b} &= \frac{x}{I_{C*}} \\ \frac{x}{I_{C*}} &= \frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{x}{I_{KF}}} \right\} \end{aligned} \right. \tag{17}$$

I_{KF} extraction strategy (3/4)

- That leads to

$$x = I_{C^*} \cdot \left\{ 1 + \frac{I_{C^*}}{I_{KF}} \right\} \quad (18)$$

- from (13), (14), (15) and (17) we can write

$$I_{CN^*} = \frac{I_{C^*}}{I_{CL^*}} = \frac{\frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{q_b^*}}{\frac{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}{q_b^*}} = \frac{1}{q_b^*} = \frac{I_{C^*}}{x} \quad (19)$$

- by substituting x in (19) by its value (18), we obtain the final expression of I_{CN^*} vs. I_{C^*}

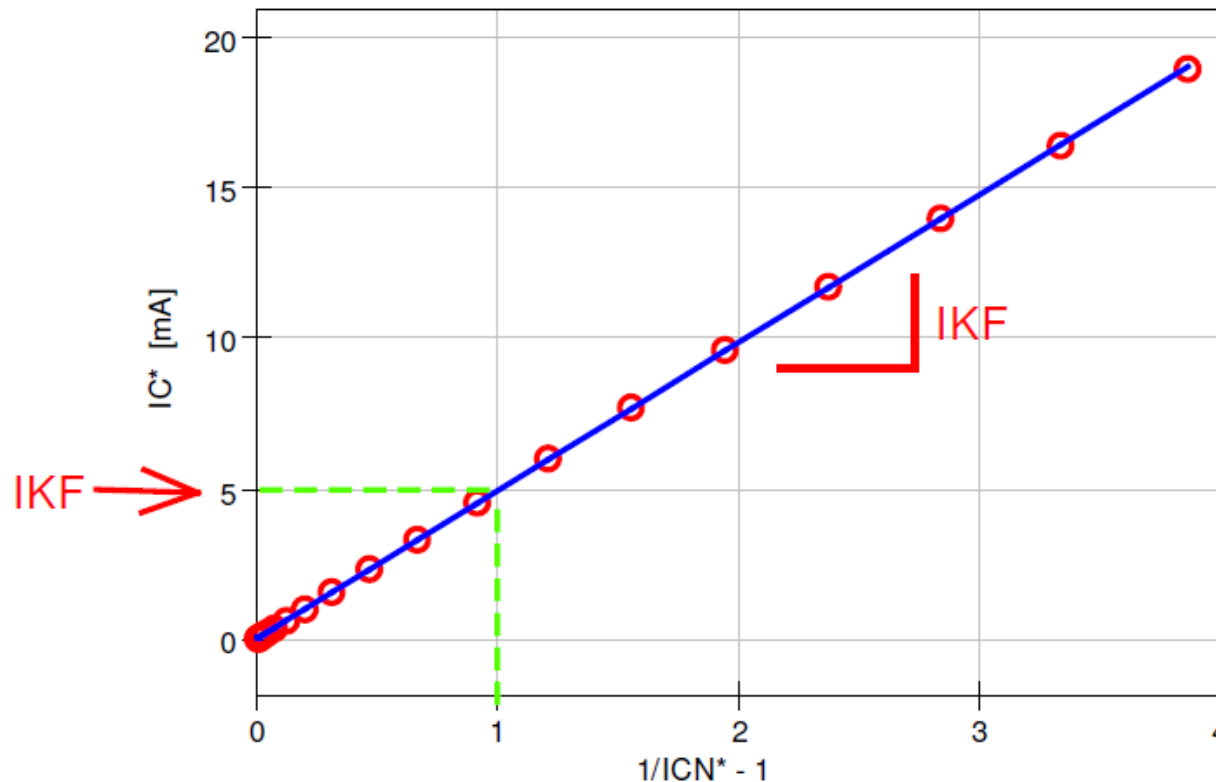
$$I_{CN^*} = \frac{I_{C^*}}{I_{C^*} \cdot \left\{ 1 + \frac{I_{C^*}}{I_{KF}} \right\}} = \frac{1}{1 + \frac{I_{C^*}}{I_{KF}}} \quad (20)$$

- This equation can be rewritten

$$I_{C^*} = I_{KF} \cdot \left\{ \frac{1}{I_{CN^*}} - 1 \right\} \quad (21)$$

I_{KF} extraction strategy (4/4)

- Equation (21) is very interesting and demonstrates that
 - The collector current (corrected from the reverse Early voltage) I_C^* is a linear function of $1/I_{CN}^* - 1$
 - I_C^* vs. $1/I_{CN}^* - 1$ is independent of series resistances
 - Its slope is equal to I_{KF}
 - $I_C^* = I_{KF}$ for $1/I_{CN}^* - 1 = 1$



■ I_{KF} (R_E) extraction flow

- Estimation of the internal V_{BEi} from the base current

$$V_{BEi} = V_T \cdot \ln \left\{ \frac{B_F \cdot I_B}{I_S} \right\}$$

- Correction of the collector current from the reverse Early voltage V_{AR}

$$I_{C^*} = \frac{I_C}{1 - \frac{V_{BEi}}{V_{AR}}}$$

- Calculation of the normalized collector current I_{CN^*}

$$I_{CN^*} = \frac{I_{C^*}}{I_S \cdot e^{\frac{V_{BEi}}{V_T}}}$$

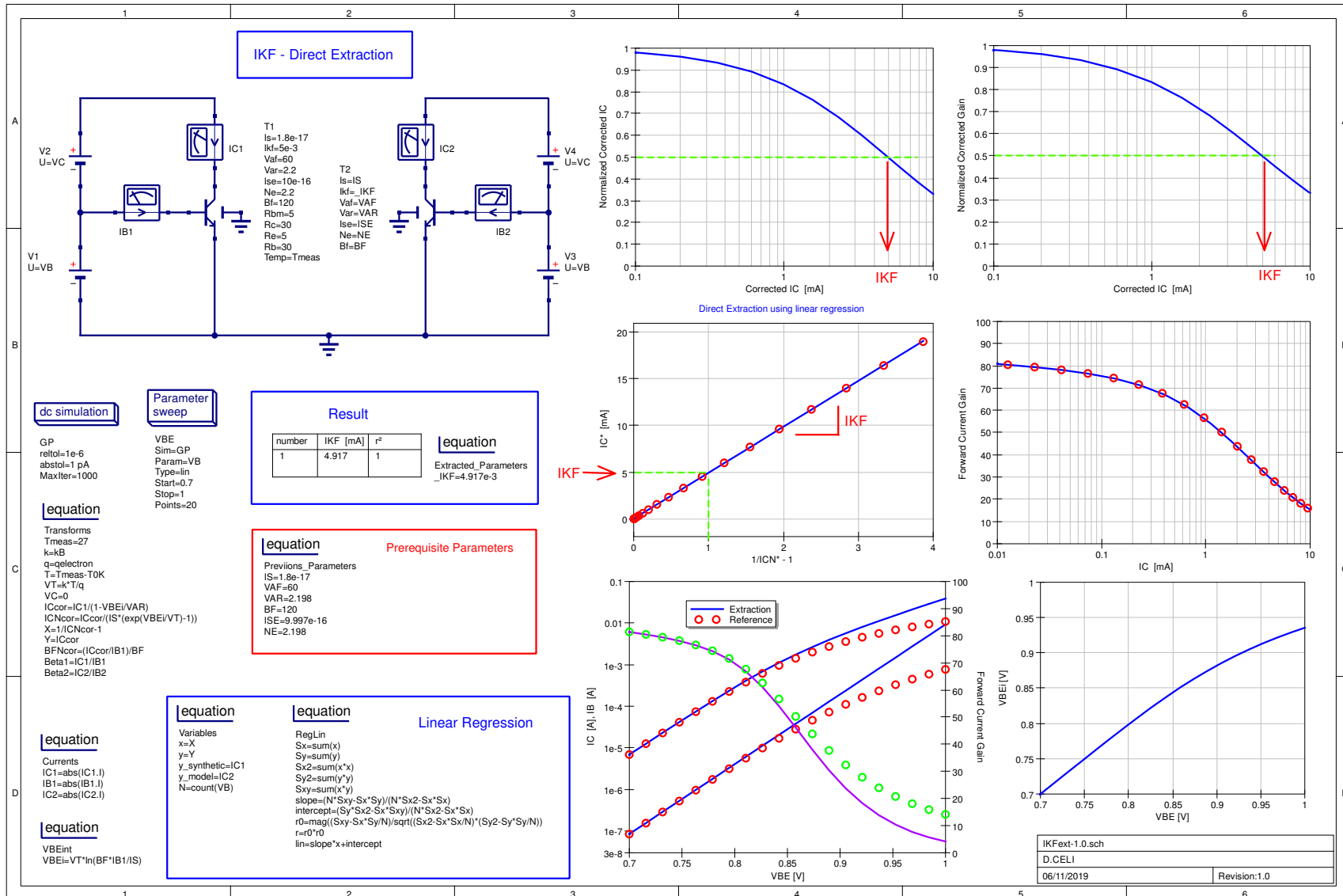
- plot I_{C^*} vs. $1/I_{CN^*} - 1$, the slope gives directly I_{KF} , without any optimization

$$I_{C^*} = I_{KF} \cdot \left\{ \frac{1}{I_{CN^*}} - 1 \right\}$$

- Once I_{KF} is known, optimization of R_E on the $I_C(V_{BE}, V_{CB}=0)$ characteristics at high-current

I_{KF} , R_E extraction flow: direct extraction of I_{KF}

Validation from synthetic data using QucsStudio worksheet



I_{KF} , R_E extraction: R_E optimization

Validation from synthetic data using QucsStudio worksheet

IKF and RE - Global Optimization

dc simulation

GP
reltol=1e-6
abstol=1 pA
MaxIter=1000

Parameter sweep

VBE
Sim=GP
Param=VBE
Type=lin
Start=0.7
Stop=1
Points=20

equation

Currents
 $IC1 = \text{abs}(IC1.I)$
 $IB1 = \text{abs}(IB1.I)$
 $IC2 = \text{abs}(IC2.I)$

equation

VBEiint
 $VBEi = VT \cdot \ln(BF \cdot IB1 / IS)$

equation

Transforms
 $Tmeas = 27$
 $k = kB$
 $q = q_{\text{electron}}$
 $T = Tmeas + T0K$
 $VT = k \cdot T / q$
 $VC = 0$
 $ICcor = IC1 / (1 - VBEi / VAR)$
 $ICNcor = ICcor / (IS \cdot \exp(VBEi / VT) - 1)$
 $X = 1 / ICNcor - 1$
 $Y = ICcor$
 $BFNcor = (ICcor / IB1) / BF$
 $Beta1 = IC1 / IB1$
 $Beta2 = IC2 / IB2$

equation

Error
 $rms_y = \text{mag}(\text{sum}((y_synthetic - y_model) / y_synthetic)^2)$
 $rel_error = 100 \cdot (y_synthetic - y_model) / y_synthetic$
 $fit = \text{max}(\log_{10}(y_synthetic / y_model)^2)$

Result

| number | _RE.opt | rms error in % |
|--------|---------|----------------|
| 1 | 5.42 | 0.000493 |

equation

Extracted_Parameters
 $_IKF = 4.917e-3$

Prerequisite Parameters

equation

Previqus_Parameters
 $IS = 1.8e-17$
 $VAR = 60$
 $VAR = 2.198$
 $BF = 120$
 $ISE = 9.997e-16$
 $NE = 2.198$

Global Optimization

Optimization

IKF_RE
 $Sim = VBE$
 $_RE = 0.1 \dots 5 \dots 100$ linear
 $rms_y = 1$ MIN

Forward Current Gain vs IC [mA]

IC [A] and IB [A] vs VBE [V]

IKF-REext-1.0.sch
D.CELI
06/11/2019 Revision: 1.0

I_{KF} extraction from measurement (1/3)

22/28

- It was the theory and now what gives the practice?...
- Similar results if assumptions slide 15 are respected
- Extraction procedure implemented and validated in QucsStudio v2.5.7
- Many improvements since what has been written in [1] thanks to the support of Z. Huszka (AMS) [2]
 - Octave function to import measured data in QucsStudio GUI.
 - Possibility to select the range of measurement (X_{min} , X_{max}) where the model parameters will be optimized.
 - Possibility of optimize measurements with one primary and one secondary sweep.
 - Multi-linear variables regression (limited to 3 variables)



Parameter Extraction with QucsStudio_v2.5.7

32th BipAK Workshop at STMicroelectronics, Crolles,
France, November 14&15 2019

Letter Session

Zoltan Huszka
31. October 2019



IKF extraction from measurement (2/3)

IKF extraction QucsStudio worksheet

IKF - Direct Extraction from measurement

V_2
 $U_2=V_C$
 V_1
 $U_1=V_B$
 IC_2
 IB_2
 T_2
 $IS=IS$
 $IKf=IKF$
 $Var=VAR$
 $ISE=ISE$
 $NE=NE$
 $BF=BF$
 $Temp=Tmeas$
 $Tnom=Tmeas$

equation
Meas_R1

equation
Boundaries
 $VBEmin=0.8$
 $VBEmax=1.1$

equation
 $VBEint$
 $VBEi=VT*\ln(BF*IB1/IS)$
 $DVBE=VBErange-VBEi$
 $Req=DVBE/IC1$

equation
 $Bias_range$
 $U_1=V_B/V_B$
 $IC1meas=U1*ICm$
 $IBmeas=U1*IBm$
 $IC1=range(ICmeas,VBEmin,VBEmax)$
 $IB1=range(IBmeas,VBEmin,VBEmax)$
 $VBErange=range(VB,VBEmin,VBEmax)$
 $IC2=abs(IC2.1)$
 $IB2=abs(IB2.1)$
 $Betam=ICm/IBm$

dc simulation
 GP
 $reltol=1e-6$
 $abstol=1\ pA$
 $MaxIter=1000$

Parameter sweep
 VBE
 $Sim=GP$
 $Param=VB$
 $Type=list$

equation
 $Transforms$
 $Tmeas=Tamb1[1]$
 $k=k_B$
 $q=q_{electron}$
 $T=Tmeas-T0K$
 $VT=k*T/q$
 $V_C=0$
 $ICcor=IC1/(1-VBEi/Var)$
 $ICNcor=ICcor/(IS*(exp(VBEi/VT)-1))$
 $X=1/ICNcor-1$
 $Y=ICcor$
 $BFNcor=(ICcor/IB1)/BF$
 $Beta1=IC1/IB1$
 $Beta2=IC2/IB2$

Result

| number | Number of points | IKF [mA] | IKF1 [mA] | r ² |
|--------|------------------|----------|-----------|----------------|
| 1 | 31 | 4.479 | 3.989 | 0.9989 |

equation
 $Extracted_Parameters$
 $IKF=3.989e-3$

equation Prerequisite Parameters
 $Prerequisite_Parameters$
 $IS=3.114e-17$
 $VAR=3.358$
 $BF=107.9$
 $ISE=1.56e-16$
 $NE=1.53$

equation Linear Regression
 $Variables$
 $x=X$
 $y=Y$
 $N=count(VBErange)$
 $RegLin$
 $Sx=sum(x)$
 $Sy=sum(y)$
 $Sx2=sum(x*x)$
 $Sy2=sum(y*y)$
 $Sxy=sum(x*y)$
 $slope=(N*Sxy-Sx*Sy)/(N*Sx2-Sx*Sx)$
 $intercept=(Sy*Sx2-Sx*Sy)/(N*Sx2-Sx*Sx)$
 $r0=mag((Sxy-Sx*Sy/N)/sqrt((Sx2-Sx*Sx/N)*(Sy2-Sy*Sy/N)))$
 $r=r0^2$
 $lin=slope*x+intercept$

Normalized Corrected IC vs Corrected IC [mA]

Direct Extraction using linear regression

IC [mA] vs 1/ICN - 1

Forward Current Gain vs IC [mA]

IC [A], IB [A] vs VBE [V]

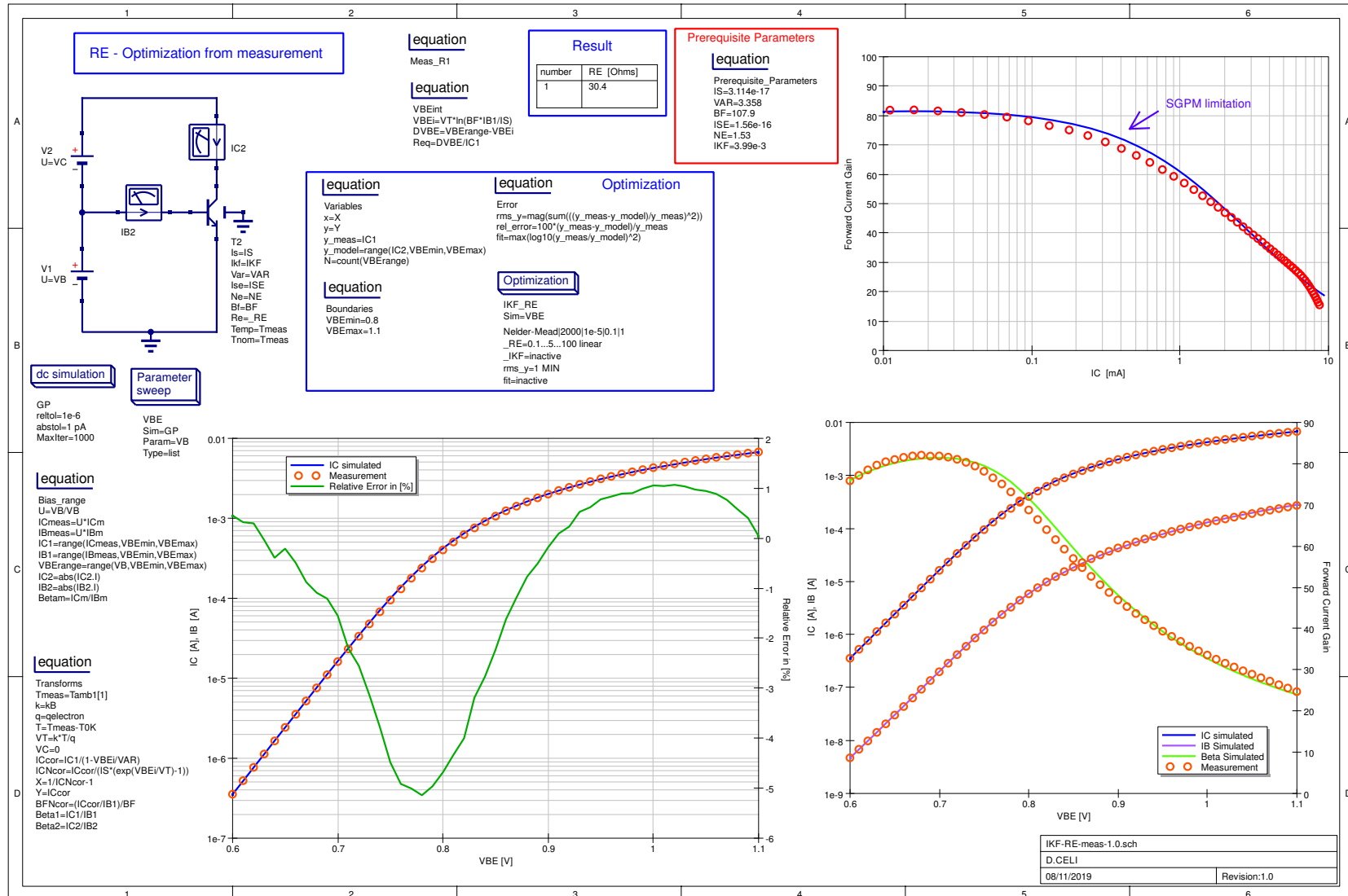
Req [Ohm] vs IC [mA]

$Req = (VBE-VBEi)/IC$

IKF-mes-1.0.sch
 D.CELI
 07/11/2019 Revision:1.0

I_{KF} extraction from measurement (3/3)

■ Optimization of R_E QucsStudio worksheet



- Development of a new method for the extraction of the knee current I_{KF} of the SGP model
- Validation of the approach from both *synthetic* and measured data with QucsStudio
- **Weakness of the I_{KF} extraction procedure**
 - The proposed method fails if
 - *Too important self-heating at high currents*
 - ⇒ in this case use lower V_{BE} range where equation (21) is linear
 - ⇒ used global optimization with the risk to have strong correlation between I_{KF} and other parameters
 - *Too important collector resistance R_C leading to the saturation of the device*
 - ⇒ Try to work at negative V_{BC} .
 - ⇒ Use global optimization with the risk to have strong correlation between I_{KF} and other parameters
 - *If the SGP model is not enough accurate to describe the behavior of the device at high currents (case of HBTs...)*
 - ⇒ Use more physics based models like HICUM.
- QucsStudio is a fantastic FOSS EDA tool, that allows in few minutes to build worksheets for the development and the validation of extraction methods. For more details see also [2] and [3].
 - You have to know the extraction method, QucsStudio will do the rest...

Appendix A: : Linear regression formula

- Linear regression is a method for calculating the equation of the *best* straight line that passes through a set of points.
- The *best* meaning the straight line that passes as closely as possible to as many points as possible.
- The best straight line equation is $y = \mathbf{a} \cdot x + \mathbf{b}$, where the slope \mathbf{a} and the intercept \mathbf{b} are given by

$$\mathbf{a} = \frac{n \cdot \sum_{i=1}^n x_i \cdot y_i - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n y_i}{n \cdot \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

$$\mathbf{b} = \frac{\sum_{i=1}^n y_i \cdot \sum_{i=1}^n x_i^2 - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n x_i \cdot y_i}{n \cdot \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

- The correlation coefficient r is given by

$$r = \frac{\sum_{i=1}^n x_i \cdot y_i - \frac{1}{n} \cdot \sum_{i=1}^n x_i \cdot \sum_{i=1}^n y_i}{\sqrt{\left\{ \sum_{i=1}^n x_i^2 - \frac{1}{n} \cdot \left(\sum_{i=1}^n x_i \right)^2 \right\} \cdot \left\{ \sum_{i=1}^n y_i^2 - \frac{1}{n} \cdot \left(\sum_{i=1}^n y_i \right)^2 \right\}}}$$

- It is a number which give you an idea if how closely the straight line fits the data. r is between +1 and -1. Values of r close to +1 or -1 indicate a good fit. Value of r close to 0 indicate a poor fit. The sign of r is linked to the sign of the slope. Therefore, sometime r^2 is used instead r to represent how well the line fits the data.

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