Basic Principle of Model Parameter Extraction - Application to the Knee Current of SGP Model with QucsStudio

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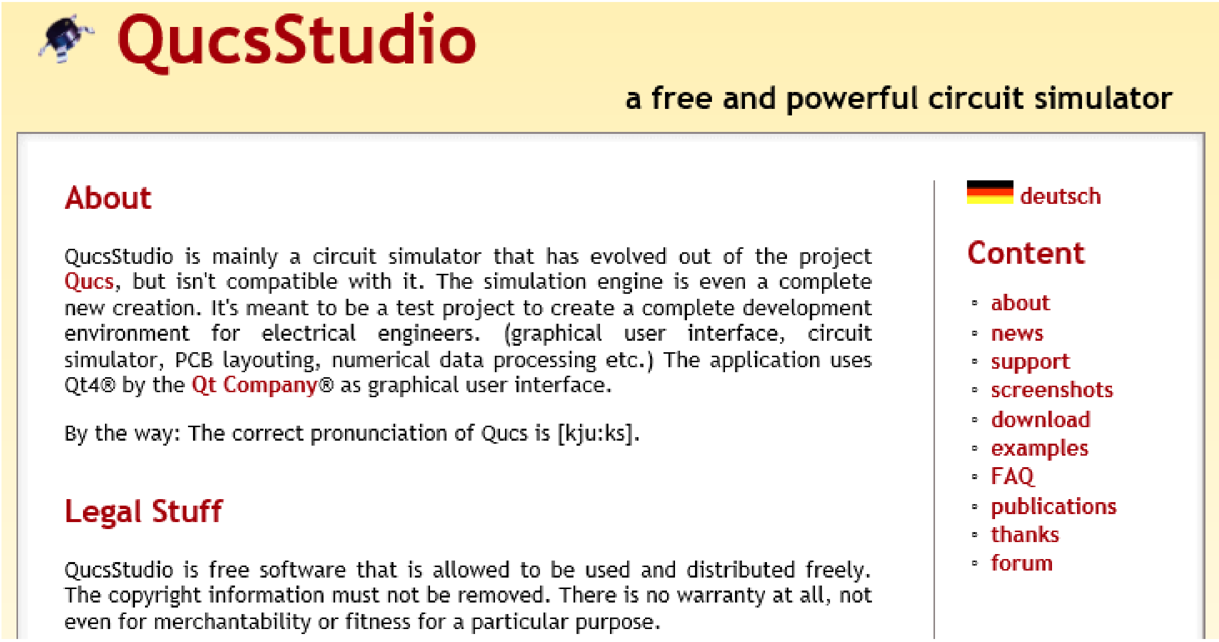
Purpose

- Reminder on the basic principles for the extraction of model parameters
- Application to the extraction of the forward knee current I_{KF} of the SPICE Gummel-Poon (SGP) model
- In complement to [1], [2] and [3] the Free and Open Source Software (FOSS) QucsStudio [4], [5] is used to implement and validate the extraction procedure



Basic principle of parameter extraction (1/2)

Objectives

- Independently of the model used, we want ***reliable*** model parameters
- ***Reliable*** meaning both ***physical*** and ***accurate*** model parameters
- *Do not forget that a physics-based model with inaccurate model parameters can be worse than a less accurate compact model but with physical model parameters*

Constraints

- All models have their own limitations
- Measurements are more or less accurate
- Therefore, how to determine model parameters both accurate and physics-based taking into account the limits of the compact models and the inaccuracy of measurements?

Key solution

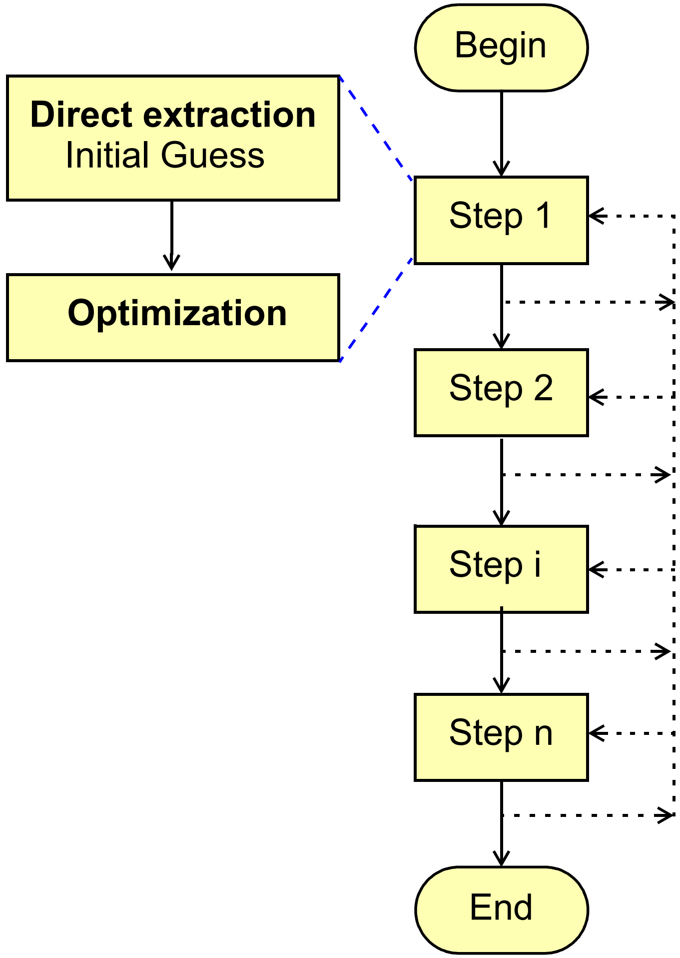
- Developing direct extraction procedures using e.g. linear regression (Appendix A) gives the solution without any iteration loop, without initial guesss and then avoids correlation between model parameters...

Advantages

- Easy parameter extraction, the only difficulty being to find the adequate transformations for linearizing the equations of the compact models, an important job of modeling engineers.
- Allows to validate both compact models and measurements.
  - If the theory predict that a given characteristic must be linear and if the measurements are also linear, that validates both the measured data and the model equations.
  - If it was not the case, that allows to alert the modeling engineers: either it is a model limitation or a measure-ments issue (limitation of the equipments, wrong test structures or measurement setup), or both.
    - ⇒ In this case accurate extraction of model parameters will be not possible.

Basic principle of parameter extraction (2/2)

- The parameter extraction is performed in several steps
  - With possible loops between the different steps
- At each extraction step
  - a given set of model parameters is determined
  - from electrical characteristics (DC, AC, noise, temperature) where the set of extracted parameters have the most impact.
- Each step is divided in 2 parts
  - The first part consists of a direct extraction of the model parameters (initial guess).
  - The second part uses non-linear least-squares algorithms for the determination of the parameters with initial guess coming from the first part.

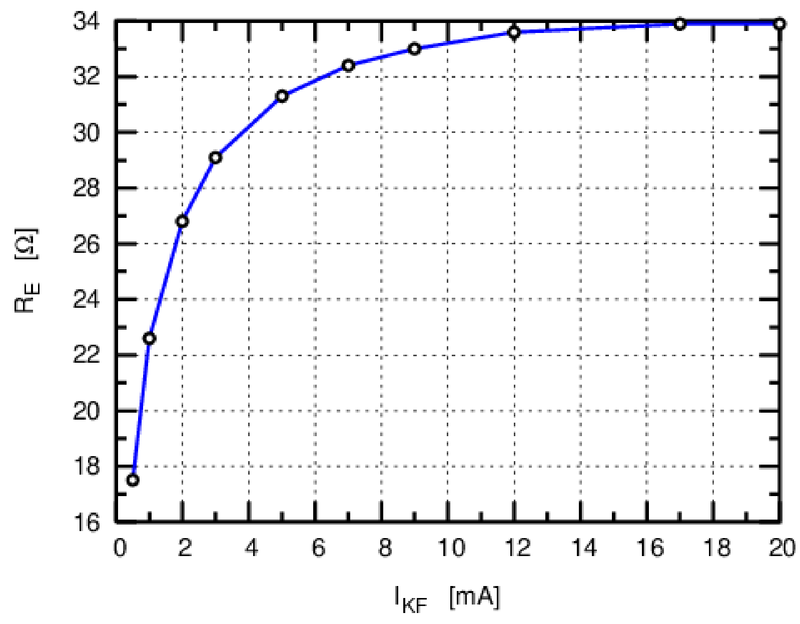


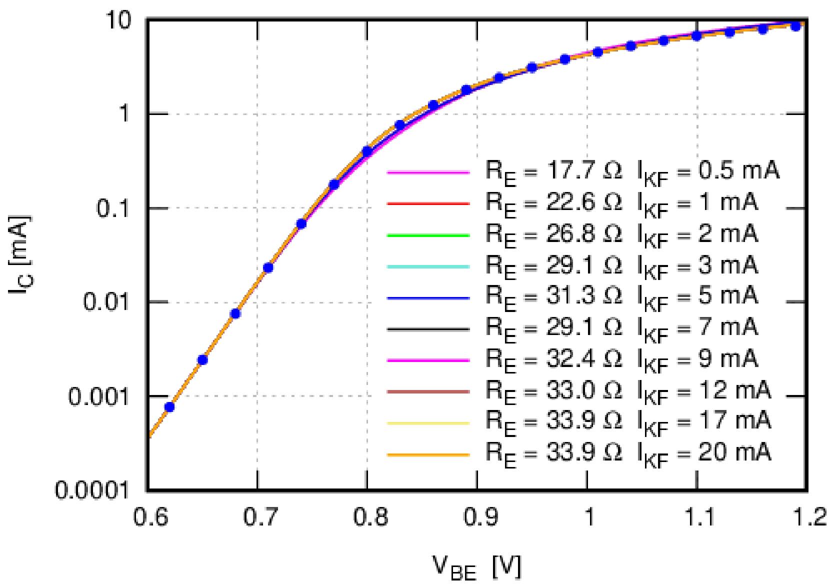
Application to the knee current I_{KF} of the SGP model

- Why to choose $\mathbf{I_{KF}}$ as example?

- Because it is a typical case where global optimization could give unrealistic $\mathbf{I_{KF}}$ values depending on the values of the emitter resistance $\mathbf{R_E}$.

- From measurement, by optimizing the collector current I_C at high-current, several ($\mathbf{I_{KF}}$, $\mathbf{R_E}$) combinations give similar *fit*.





I_{KF} explained (1/3)

- Why  current and for what?

- In SGP model, the forward knee current $\mathbf{I_{KF}}$ is used to model the high-injection effects
  - High-injection effects occur when injected minority carriers are greater that the doping level.

- Model formulation
  - Forward mode ($V_{BEi} > 0$ and $V_{BCi} = 0$ V)
  - No Early effect $\mathbf{V_{AF}} = \mathbf{V_{AR}} = \infty$
  - The collector current can be written

$$I_C = \frac{\mathbf{I_S}}{q_b} \cdot \left(e^{\frac{V_{BEi}}{V_T}} - 1 \right) \approx \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{q_1}{2} \cdot \{1 + \sqrt{1 + 4q_2}\}} \quad \text{with} \qquad (1)$$

$$q_1 = \frac{1}{1 - \frac{V_{BEi}}{\mathbf{V_{AR}}} - \frac{V_{BCi}}{\mathbf{V_{AF}}}} \approx 1 \quad \text{Early effects} \qquad (2)$$

$$q_2 = \frac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot \left(e^{\frac{V_{BEi}}{V_T}} + 1 \right) + \frac{\mathbf{I_S}}{\mathbf{I_{KR}}} \cdot \left(e^{\frac{V_{BCi}}{V_T}} + 1 \right) \approx \frac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot e^{\frac{V_{BEi}}{V_T}} \quad \text{High-current effects} \qquad (3)$$

I_{KF} explained (2/3)

- From (1), (2) and (3) the collector current in forward mode can be written

$$I_C = \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} \qquad (4)$$

- Asymptotic value at low currents $I_C << \mathbf{I_{KF}}$

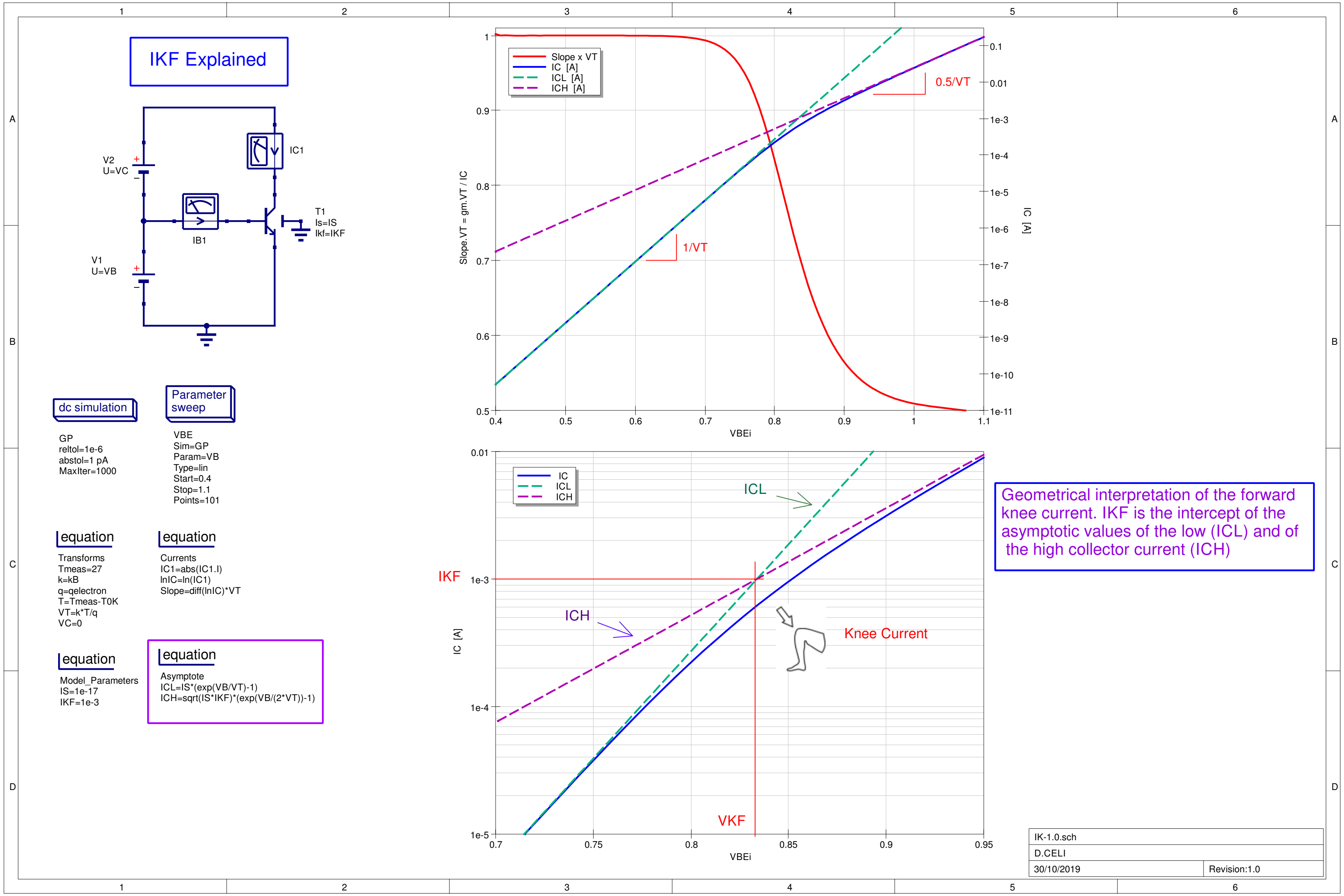
$$I_{CL} = \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} \approx \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \{1 + \sqrt{1}\}} = \mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}} \qquad (5)$$

- Asymptotic value at high current $I_C >> \mathbf{I_{KF}}$

$$I_{CH} = \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \frac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} \approx \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{1}{2} \cdot \left\{ \sqrt{4 \cdot \frac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\}} = \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{2 \cdot \sqrt{\frac{\mathbf{I_S}}{\mathbf{I_{KF}}}} \cdot e^{\frac{V_{BEi}}{\mathbf{2} \cdot V_T}}} = \boxed{\sqrt{\mathbf{I_S} \cdot \mathbf{I_{KF}}}} \cdot e^{\frac{V_{BEi}}{\mathbf{2} \cdot V_T}} = \boxed{\mathbf{I_{SH}}} \cdot e^{\frac{V_{BEi}}{\mathbf{2} \cdot V_T}} \qquad (6)$$

I_{KF} explained (3/3)

- QucsStudio worksheet for $\mathbf{I_{KF}}$ explanation

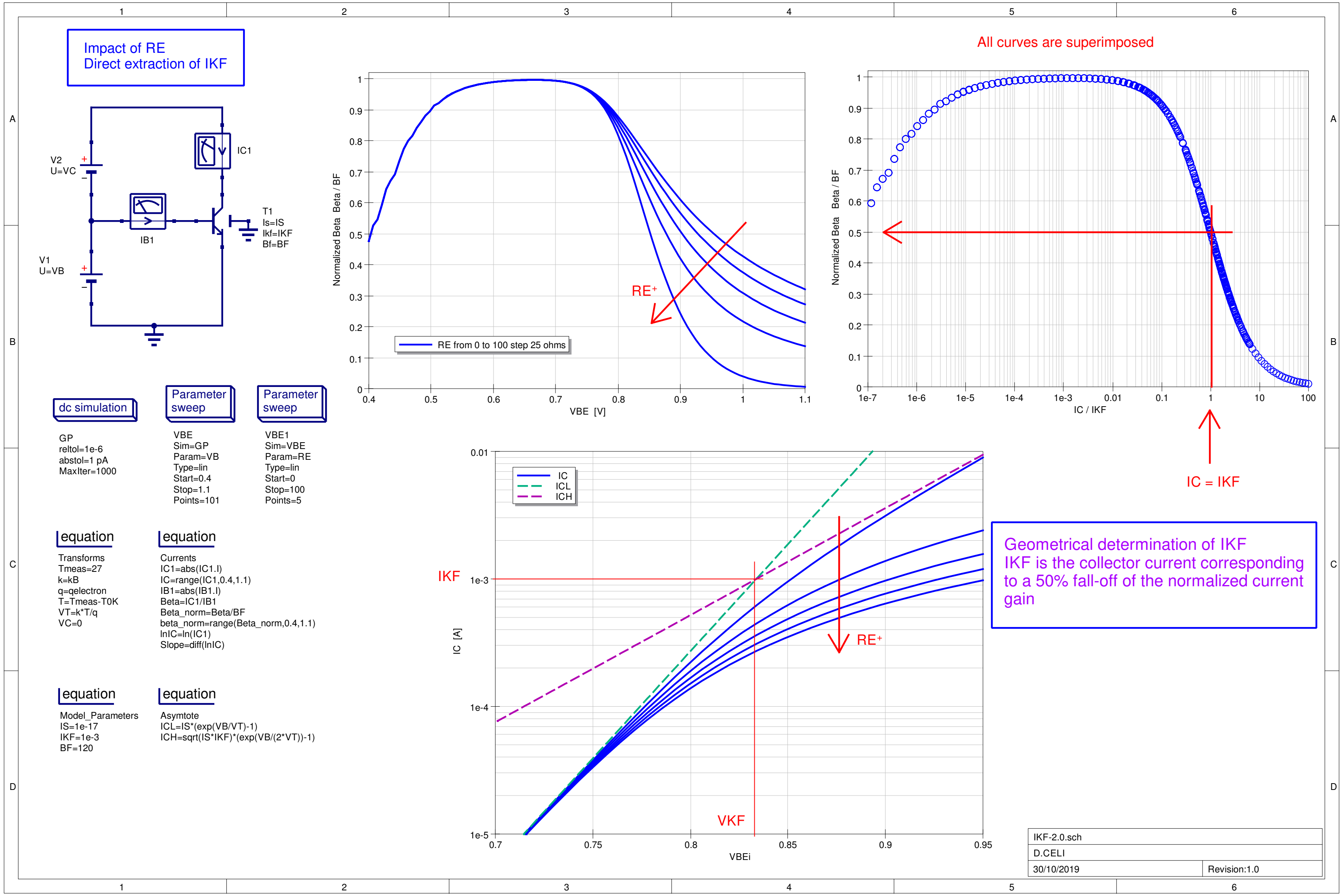


I_{KF} summary

- **I_{KF}** affects the bending of I_C at high V_{BE}
- And therefore the current gain β fall-off at high-current
  - $\beta = I_C/I_B$, I_B not affected by high-injection effects as the doping level of the emitter is too high.
- But unfortunately, as already shown in slide 4, other important parasitic effects also affect the curvature of I_C at high currents
  - Voltage drop in series resistances (R_E, R_B, R_C)
  - Self-heating (SH)
- Now, the main question is how to extract **I_{KF}** without to be impacted by these parasitic effects?
- For that, we will analyze the dependence of the normalized current gain β/**B_F**, at V_{BC}=0V, versus V_{BE} and I_C
  - Simulations performed with QucsStudio

β/B_F versus V_{BE} and I_C (1/2)

- QucsStudio worksheet impact of R_E on the forward current gain



$\beta/\mathbf{B_F}$ versus V_{BE} and I_C (2/2)

- Main important results.

- $\beta/\mathbf{B_F}$ versus V_{BE} is strongly impacted by the emitter resistance, but also by the base resistance (not shown here).

- But in the contrary, $\beta/\mathbf{B_F}$ versus I_C (@ V_{BC}=0V) is not affected by R_E and R_B, and $\mathbf{I_{KF}}$ is the value of the collector current corresponding to a 50% fall-off (at high-current) of the normalized current gain.

- This method is a direct method allowing to have a first order value of $\mathbf{I_{KF}}$, without any calculation

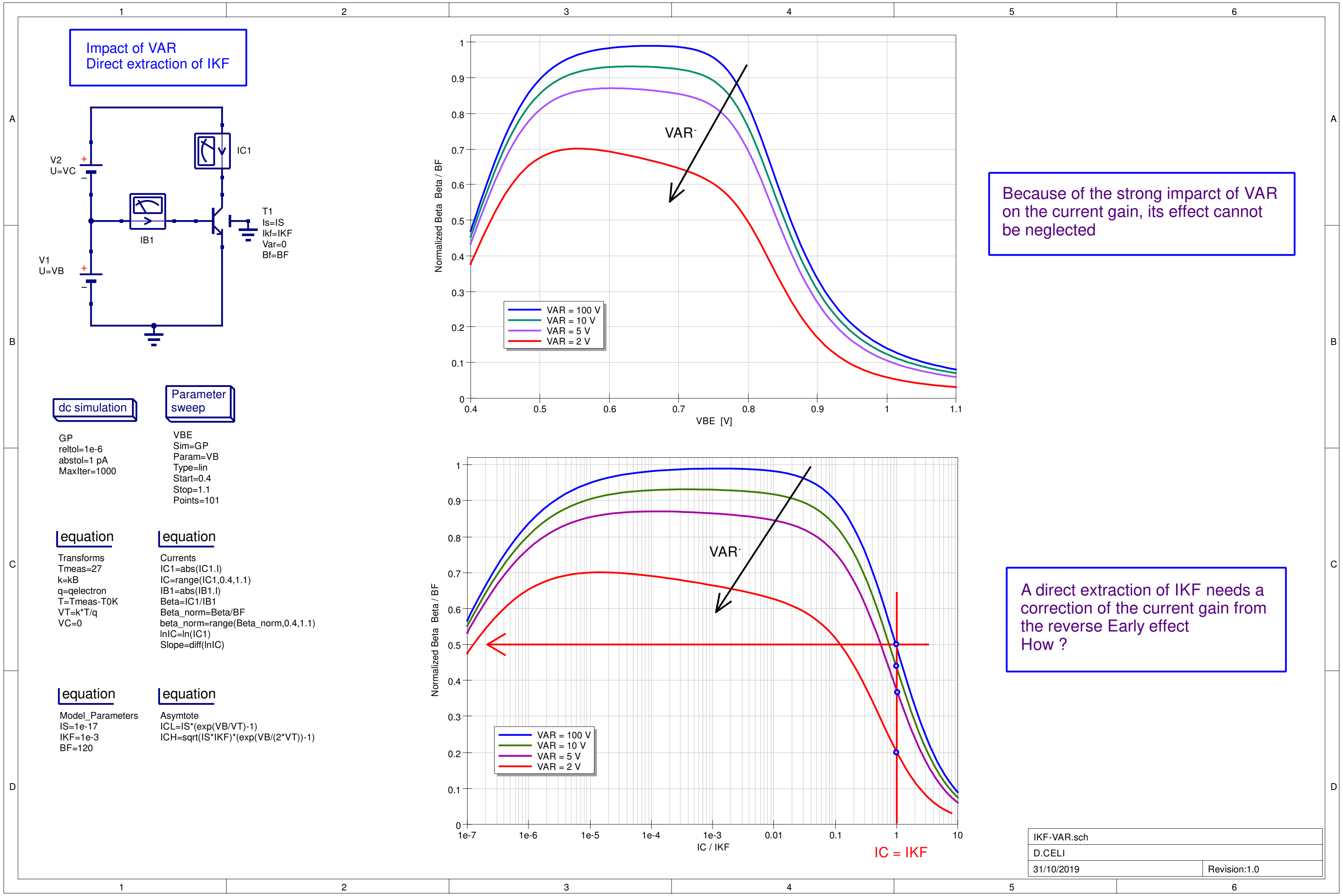
- But it is not so simple, up to now the impact of the reverse Early voltage has been neglected ($V_{AR} = \infty$, $q_1 = 1$)

$$q_1 = \frac{1}{1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}} \quad (7)$$

- This approximation is not valid for modern BJTs or HBTs

Impact of V_{AR}

- QucsStudio worksheep showing the impact of $\mathbf{V_{AR}}$ on the forward current gain



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How to correct the impact of $\mathbf{V_{AR}}$?

- Impact of V_{AR} on I_C
  - From (1) and (7) we can write

$$I_C = \frac{\mathbf{I_S}}{q_b} \cdot \left(e^{\frac{V_{BEi}}{V_T}} - 1 \right) \approx \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{\frac{q_1}{2} \cdot 1 + \sqrt{1 + 4q_2}} = \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{q_1 \cdot q^*{}_b} = \frac{\mathbf{I_S} \cdot \left\{ 1 - \frac{V_{BEi}}{\mathbf{V_{AR}}} \right\} \cdot e^{\frac{V_{BEi}}{V_T}}}{q_{b*}} \quad \text{with} \tag{8}$$

$$q_{b*} = \frac{1 + \sqrt{1 + 4q_2}}{2} \quad \text{is the normalized majority base charge without Early effect} \tag{9}$$

- Correction of I_C from the impact of $\mathbf{V_{AR}}$
  - From (8) the corrected I_{C*} is defined by

$$I_{C*} = \frac{I_C}{1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}} = \frac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{q^*{}_b} \tag{10}$$

- Therefore, the correction of I_C from the reverse Early voltage needs to know the internal base emitter voltage V_{BEi}

Estimation of V_{BEi}

- How to estimate V_{BEi} without to know the access series resistances R_E and R_B?
- Assumption
  - We assume that at high V_{BE} and V_{BC} = 0V the base recombination current is negligible and that we can write

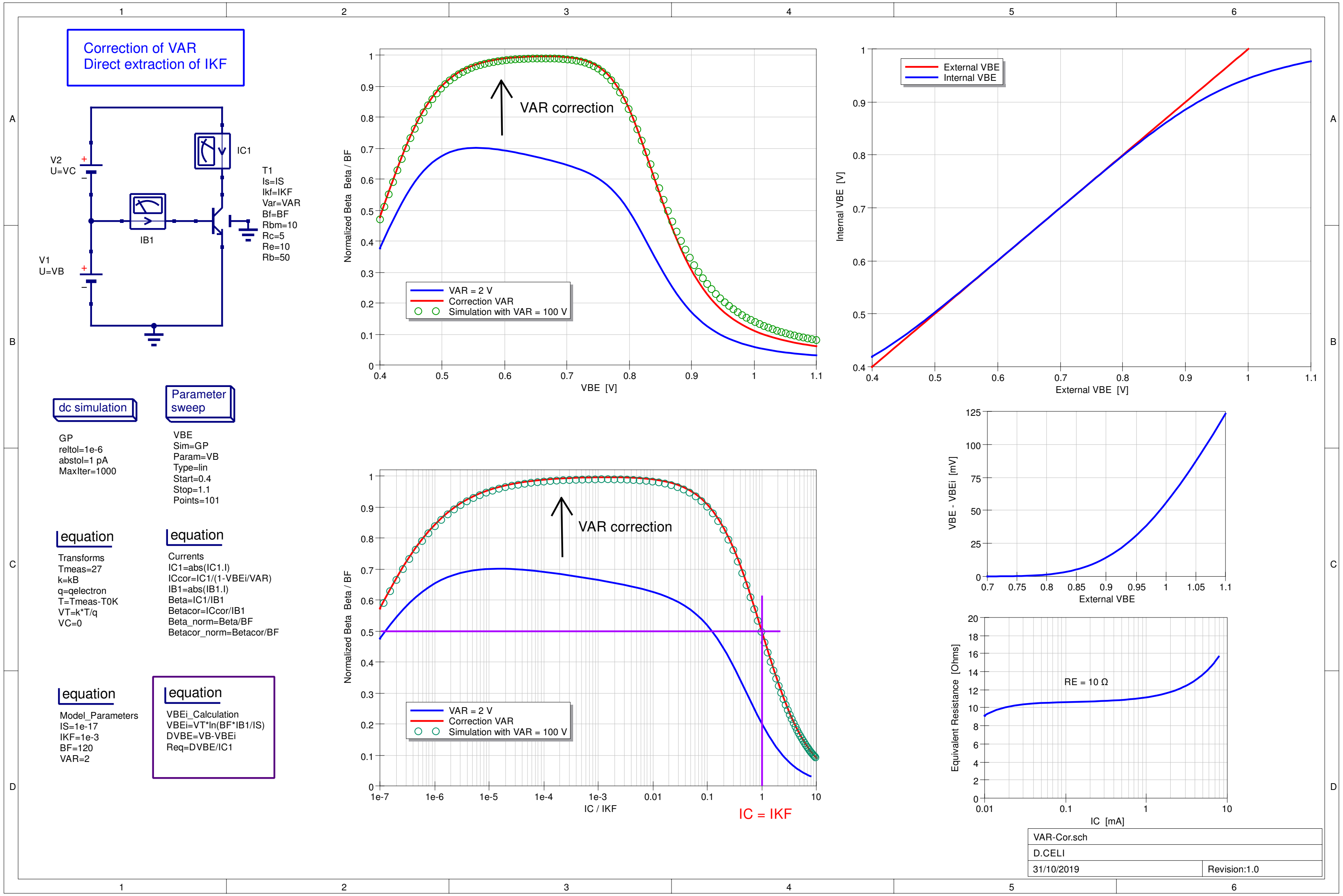
$$I_B = \frac{\mathbf{I_S}}{\mathbf{B_F}} \cdot e^{\frac{V_{BEi}}{V_T}} \tag{11}$$

- V_{BEi} calculation
  - Knowing $\mathbf{I_S}$ and $\mathbf{B_F}$, from (11), it is easy to compute V_{BEi}

$$V_{BEi} = V_T \cdot \ln\left\{\frac{\mathbf{B_F} \cdot I_B}{\mathbf{I_S}}\right\} \tag{12}$$

VAR correction

- QucsStudio worksheet correction of V_{AR}



I_{KF} extraction strategy (1/4)

- From all these previous observations, we will be able to define an extraction strategy for $\mathbf{I_{KF}}$ without to have to know the values of the access series resistances

- Assumptions
  - SGP model (with its limitations)
  - No (or negligible) self-heating
  - Sufficient low collector resistance $\mathbf{R_C}$ to avoid the saturation of the device at V_{BC} = 0V and at high-current

- Prerequisite model parameters
  - Low collector current parameters: $\mathbf{I_S}$
  - Base current parameters: $\mathbf{B_F}$, $\mathbf{I_{SE}}$, $\mathbf{N_E}$
  - Reverse Early voltage $\mathbf{V_{AR}}$

- Comments
  - In slide 9 we have observed that the forward current gain, corrected from V_{AR}, $\beta^* = I_{C*}/I_B$ vs. I_{C*} was independent of the series resistances. I_{C*} is given by (10).
  - The idea is not to use β^*, to be not impacted by the possible *non-ideality* of I_B, but the normalized collector current $I_{CN*} = I_{C*}/I_{CL*}$, where I_{CL*} is simply defined by

$$I_{CL*} = \mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}} \tag{13}$$

I_{KF} extraction strategy (2/4)

Expression of I_{CN*} vs. I_{C*}

- By definition the normalized collector current is given by

$$I_{CN*} = \frac{I_{C*}}{I_{CL*}} \tag{14}$$

- with I_{CL*} given by (13) and

$$\begin{cases} I_{C*} = \dfrac{I_C}{1 - \dfrac{V_{BEi}}{\mathbf{V_{AR}}}} = \dfrac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{q*_b} \\ q*_b = \dfrac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \dfrac{\mathbf{I_S}}{\mathbf{I_{KF}}} \cdot e^{\frac{V_{BEi}}{V_T}}} \right\} \end{cases} \tag{15}$$

- We want a formulation of I_{C*} independent of V_{BEi}, let us write

$$x = \mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}} \tag{16}$$

$$(15) \Leftrightarrow \begin{cases} q*_b = \dfrac{x}{I_{C*}} \\ \dfrac{x}{I_{C*}} = \dfrac{1}{2} \cdot \left\{ 1 + \sqrt{1 + 4 \cdot \dfrac{x}{\mathbf{I_{KF}}}} \right\} \end{cases} \tag{17}$$

I_{KF} extraction strategy (3/4)

- That leads to

$$x = I_{C*} \cdot \left\{ 1 + \frac{I_{C*}}{\mathbf{I_{KF}}} \right\} \tag{18}$$

- from (13), (14), (15) and (17) we can write

$$I_{CN*} = \frac{I_{C*}}{I_{CL*}} = \frac{\dfrac{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}{q*_b}}{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}} = \frac{1}{q*_b} = \frac{I_{C*}}{x} \tag{19}$$

- by substituting x in (19) by its value (18), we obtain the final expression of I_{CN*} vs. I_{C*}

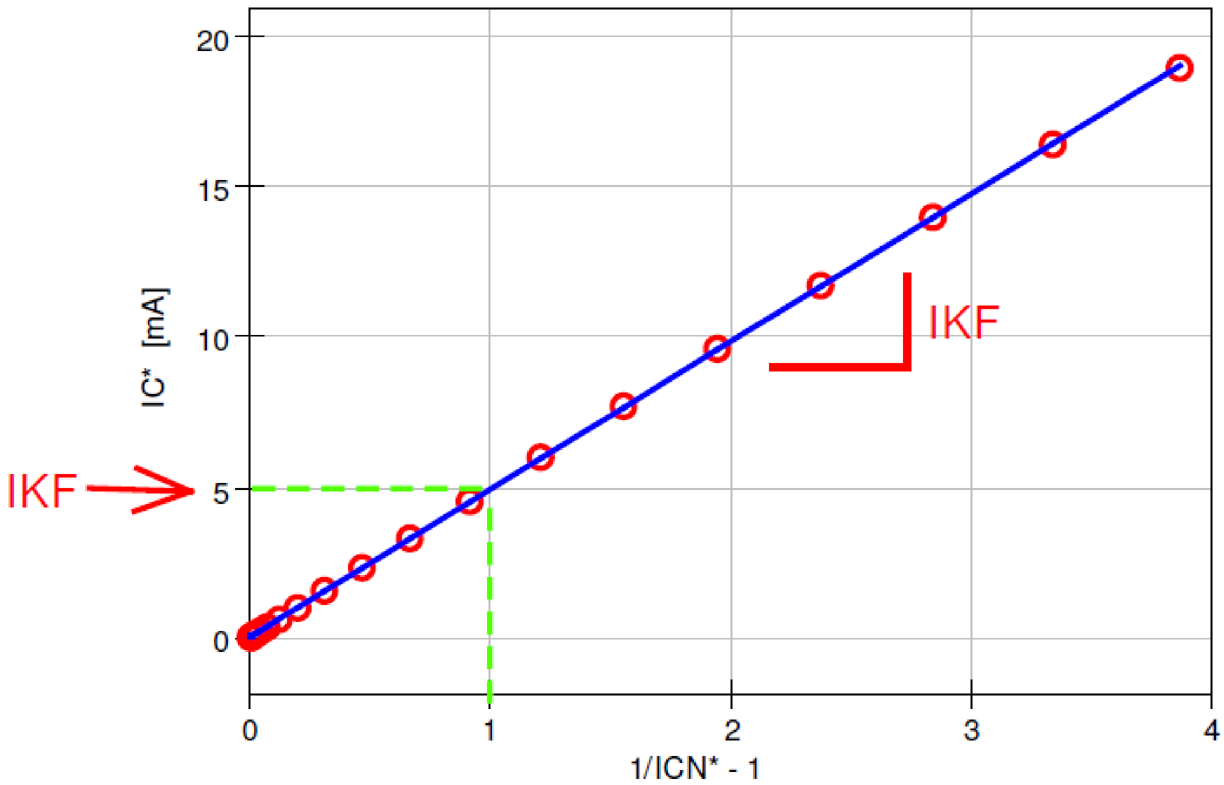
$$I_{CN*} = \frac{I_{C*}}{I_{C*} \cdot \left\{ 1 + \dfrac{I_{C*}}{\mathbf{I_{KF}}} \right\}} = \frac{1}{1 + \dfrac{I_{C*}}{\mathbf{I_{KF}}}} \tag{20}$$

- This equation can be rewritten

$$I_{C*} = \mathbf{I_{KF}} \cdot \left\{ \frac{1}{I_{CN*}} - 1 \right\} \tag{21}$$

I_{KF} extraction strategy (4/4)

- Equation (21) is very interesting and demonstrates that
  - The collector current (corrected from the reverse Early voltage) $I_C{}^*$ is a linear function of $1/I_{CN^*} - 1$
  - I_{C^*} vs. $1/I_{CN^*} - 1$ is independent of series resistances
  - Its slope is equal to $\mathbf{I_{KF}}$
  - $I_{C^*} = \mathbf{I_{KF}}$ for $1/I_{CN^*} - 1 = 1$



I_{KF} extraction flow

$\mathbf{I_{KF}}$ ($\mathbf{R_E}$) extraction flow

- Estimation of the internal V_{BEi} from the base current

$$V_{BEi} = V_T \cdot \ln\left\{\frac{\mathbf{B_F} \cdot I_B}{\mathbf{I_S}}\right\}$$

- Correction of the collector current from the reverse Early voltage $\mathbf{V_{AR}}$

$$I_{C*} = \frac{I_C}{1 - \frac{V_{BEi}}{\mathbf{V_{AR}}}}$$

- Calculation of the normalized collector current I_{CN}^*

$$I_{CN*} = \frac{I_{C*}}{\mathbf{I_S} \cdot e^{\frac{V_{BEi}}{V_T}}}$$

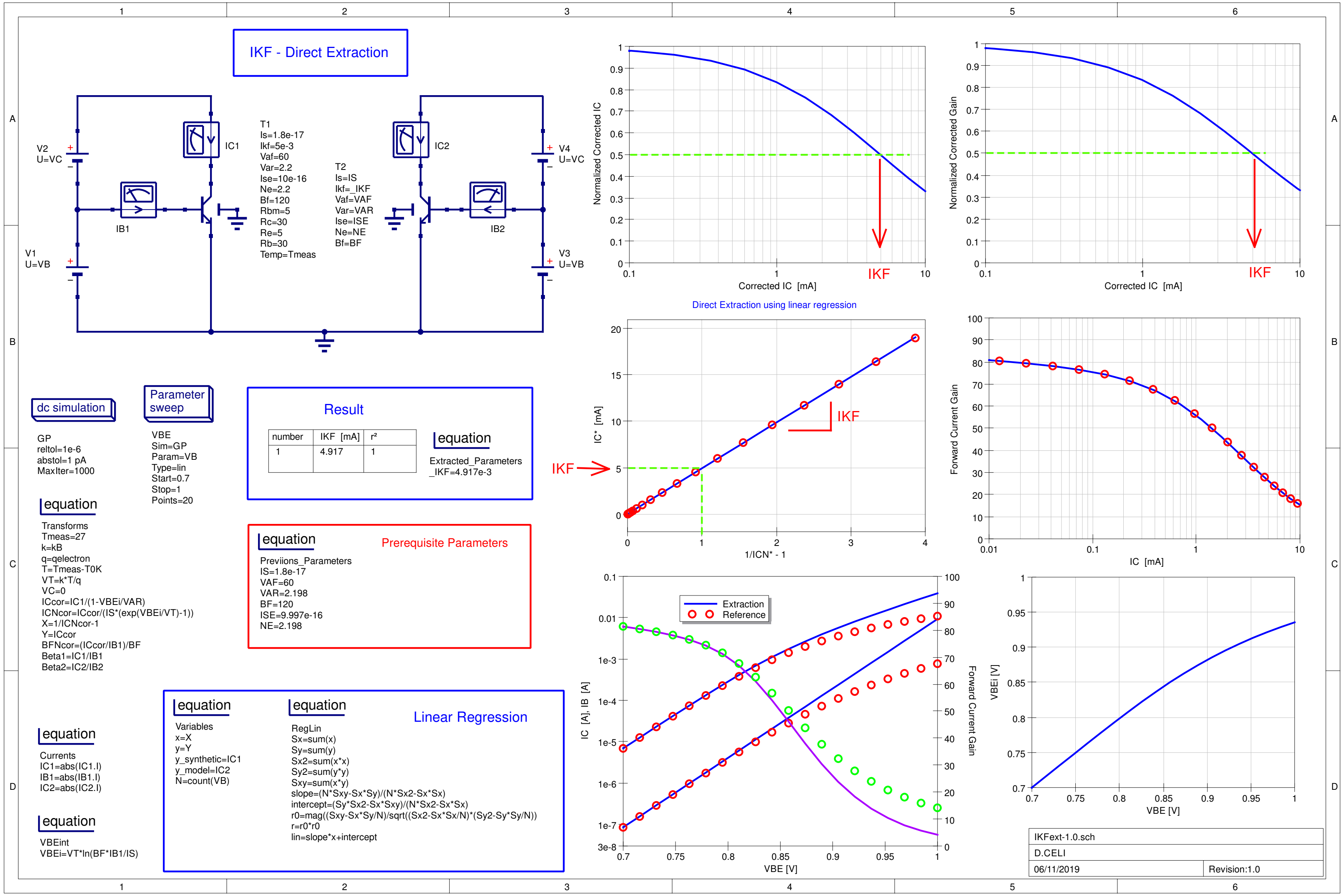
- plot I_{C*} vs. $1/I_{CN*} - 1$, the slope gives directly $\mathbf{I_{KF}}$, without any optimization

$$I_{C*} = \mathbf{I_{KF}} \cdot \left\{\frac{1}{I_{CN*}} - 1\right\}$$

- Once $\mathbf{I_{KF}}$ is known, optimization of $\mathbf{R_E}$ on the $I_C(V_{BE},V_{CB}=0)$ characteristics at high-current

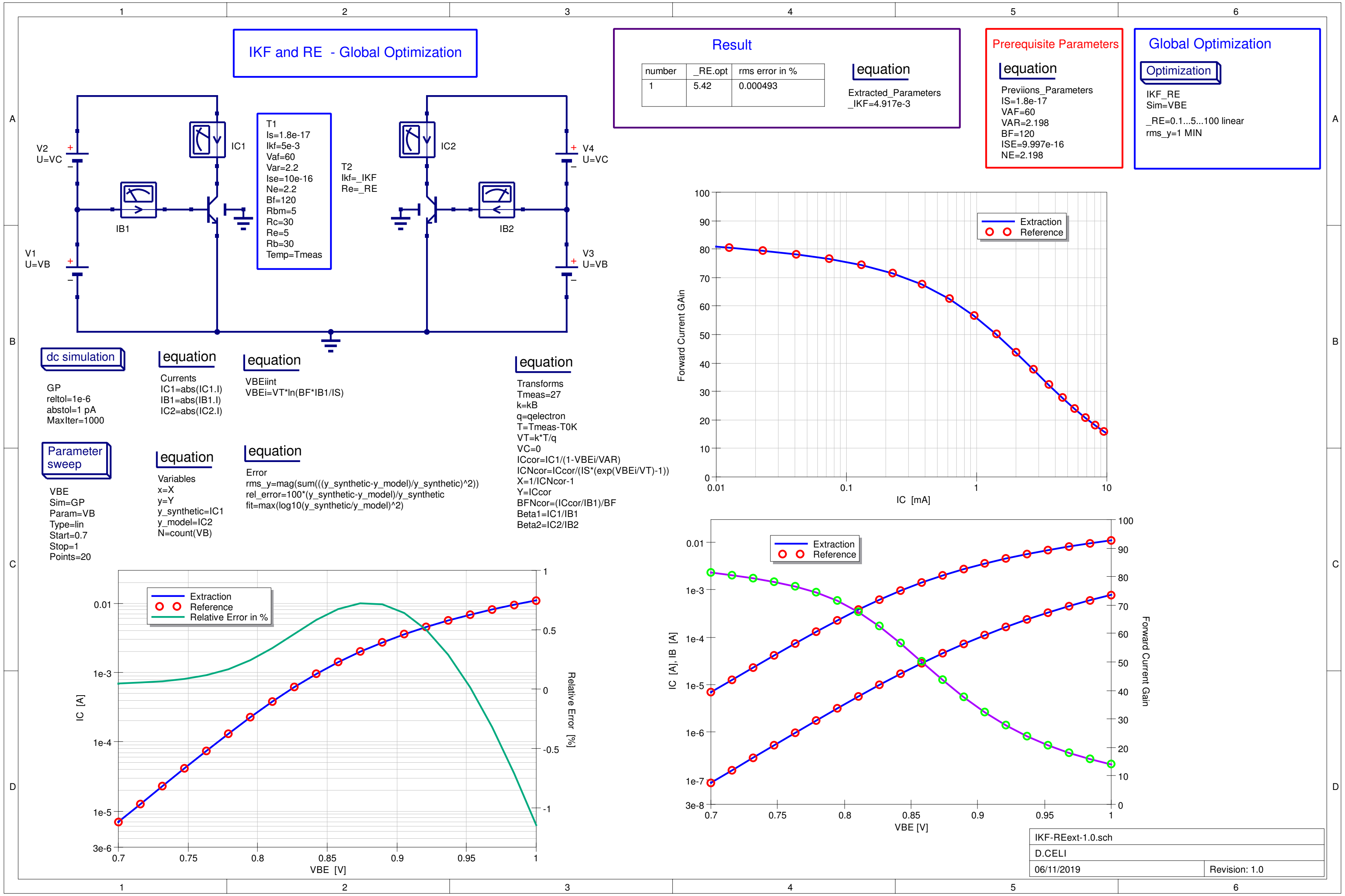
I_{KF}, R_E extraction flow: direct extraction of I_{KF}

- Validation from *synthetic* data using QucsStudio worksheet



I_{KF}, R_E extraction: R_E optimization

- Validation from *synthetic* data using QucsStudio worksheet

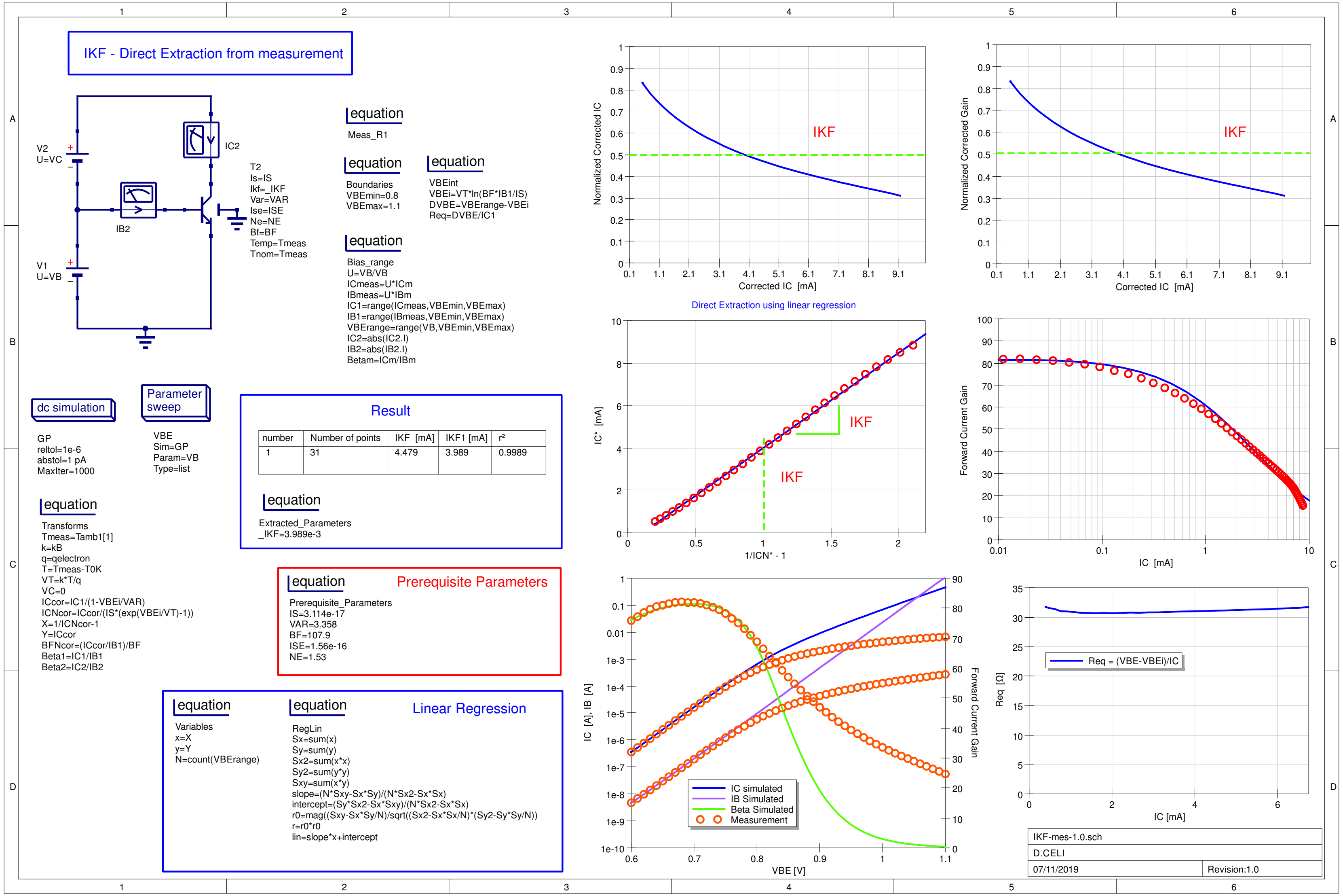


I_{KF} extraction from measurement (1/3)

- It was the theory and now what gives the practice?...
- Similar results if assumptions slide 15 are respected
- Extraction procedure implemented and validated in QucsStudio v2.5.7
- Namy improvements since what has been written in [1] thanks to the support of Z. Huszka (AMS) [2]
  - Octave function to import measured data in QucsStudio GUI.
  - Possibility to select the range of measurement (Xmin, Xmax) where the model parameters will be optimized.
  - Possibility of optimize measurements with one primary and one secondary sweep.
  - Multi-linear variables regression (limited to 3 variables)

I_{KF} extraction from measurement (2/3)

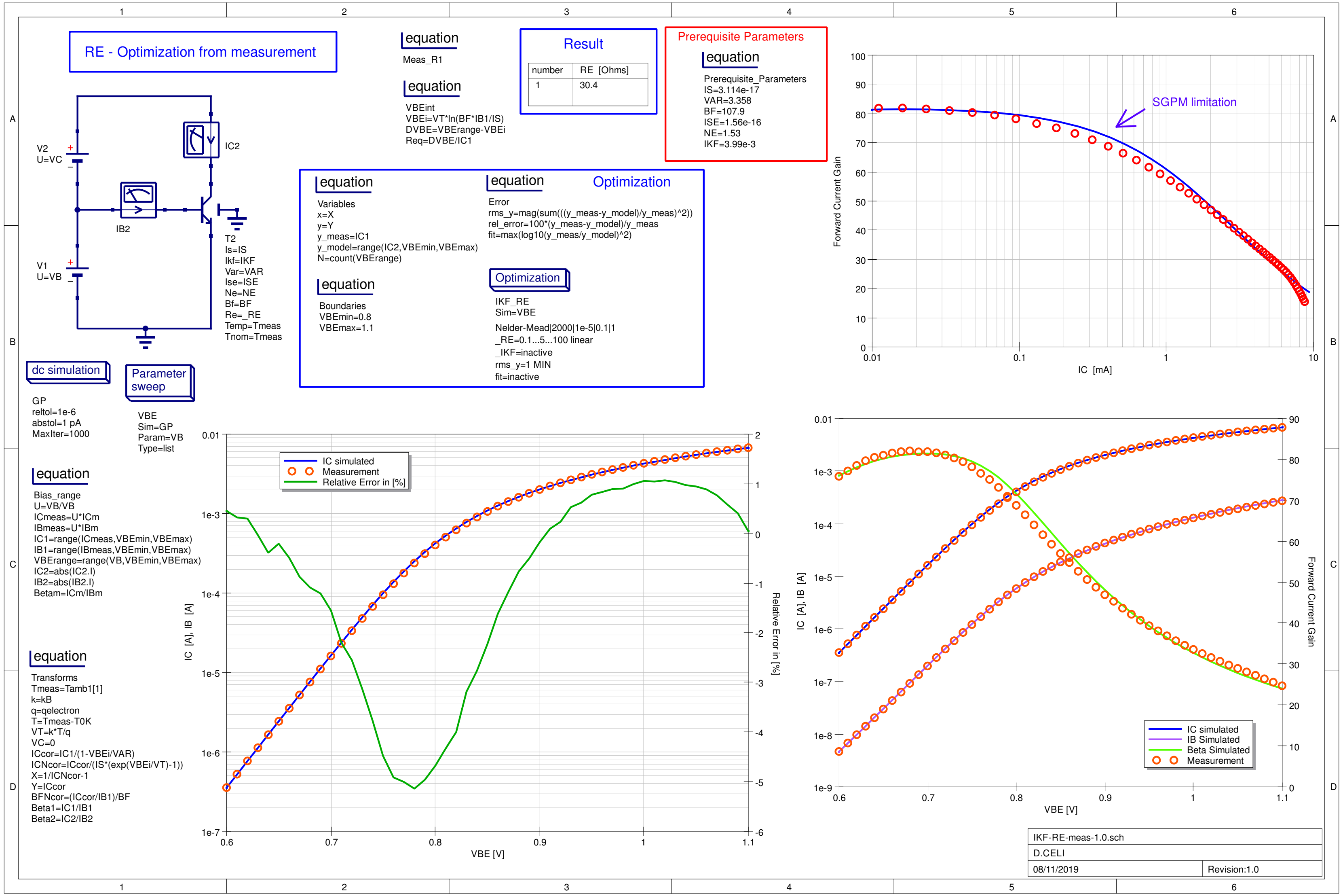
- I_{KF} extraction QucsStudio worksheet



number	Number of points	IKF [mA]	IKF1 [mA]	r²
1	31	4.479	3.989	0.9989

I_{KF} extraction from measurement (3/3)

- Optimization of $\mathbf{R_E}$ QucsStudio worksheet



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Summary

- Development of a new method for the extraction of the knee current $\mathbf{I_{KF}}$ of the SGP model

- Validation of the approach from both *synthetic* and measured data with QucsStudio

- Weakness of the I_{KF} extraction procedure
  - The proposed method fails if
    - *Too important self-heating at high currents*

      ⇒ in this case use lower V_{BE} range where equation (21) is linear

      ⇒ used global optimization with the risk to have strong correlation between I_{KF} and other parameters
    - *Too important collector resistance R_C leading to the saturation of the device*

      ⇒ Try to work at negative V_{BC}.

      ⇒ Use global optimization with the risk to have strong correlation between I_{KF} and other parameters
    - *If the SGP model is not enough accurate to describe the behavior of the device at high currents (case of HBTs...)*

      ⇒ Use more physics based models like HICUM.

- QucsStudio is a fantastics FOSS EDA tools, that allows in few minutes to build work-sheets for the development and the validation of extraction methods. For more details see also [2] and [3].
  - You have to know the extraction method, QucsStudio will do the rest...

Appendix A: : Linear regression formula

- Linear regression is a method for calculating the equation of the *best* straight line that passes through a set of points.
- The *best* meaning the straight line that passes as closely as possible to as many points as possible.
- The best straight line equation is y = **a**.x + **b**, where the slope **a** and the intercept **b** are given by

$$a = \frac{n \cdot \sum_{i=1}^{n} x_i \cdot y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2}$$

$$b = \frac{\sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i \cdot y_i}{n \cdot \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2}$$

- The correlation coefficient r is given by

$$r = \frac{\sum_{i=1}^{n} x_i \cdot y_i - \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{\sqrt{\left\{\sum_{i=1}^{n} x_i^2 - \frac{1}{n} \cdot \left(\sum_{i=1}^{n} x_i\right)^2\right\} \cdot \left\{\sum_{i=1}^{n} y_i^2 - \frac{1}{n} \cdot \left(\sum_{i=1}^{n} y_i\right)^2\right\}}}$$

- It is a number which give you an idea if how closely the straight line fits the data. **r** is between +1 and -1. Values of **r** close to +1 or -1 indicate a good fit. Value of **r** close to 0 indicate a poor fit. The sign of **r** is linked to the sign of the slope. Therefore, sometime **r²** is used instead **r** to represent how well the line fits the data.

References

[1] D. Céli, *"SGP Model parameter Extraction with QucsStudio - A First Trial..."*, 19th HICUM Workshop, Letter Session, Munich, May 13/14, 2019.

[2] Z. Huszka, *"Parameter Extraction with QucsStudio_v2.5.7"*, 32th BipAK Workshop, Letter session, Crolles, November 14/15, 2019.

[3] Z. Huszka, *"A Solution to the Q_{P0} Issue in HICUM Parameter Extraction"*, 32th BipAK Workshop, Letter session, Crolles, November 14/15, 2019.

[4] M. Margraf, "*QucsStudio - A free and powerful circuit simulator*", http://dd6um.darc.de/QucsStudio/about.html.

[5] QucsStudio Forum, *http://qucsstudio.xobor.de/*