Removing the $I_{CK}$ implied bumps in HICUM

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Letter Session
Outline

Removing the $I_{CK}$ implied bumps in HICUM

- critical parameters in the BJT models
- the generic critical current equation used in HICUM
- low and high $V_{ce}$ asymptotes
- $V_{ciei}$ masking
- implications on the forward output and FT characteristics
- the logexp smoothing
- comparision to hyperbolic smoothing
- examples
- summary
In the Ebers-Moll (EM) and Gummel-Poon (GP) BJT models only the DC high current behaviour was affected by a dedicated parameter.

**High-current parameter**

\[ I_c = I_s \frac{1}{1 + \Theta \cdot \exp\left(\frac{V_{BE}}{2kT}\right)} \]

\[ \Theta = kT \frac{V_{ifwd}}{V_{BE}} \]

\[ i_{Tf} = \left(1 - \frac{V_{BE}}{\text{var}}\right) \cdot \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{ifwd}{\text{ikf}}}}} \]

Both formulation converge to the \( \frac{1}{2} \) slope at elevated currents as predicted by Webster. The AC behaviour is independently formulated without any link to the DC domain.
In the HICUM model the critical current $I_{CK}$ has an influence both on the DC and AC characteristics through the ratio $I_{CK}/I_C$. The generic formulation of this parameter [1]:

$$I_{CK} = \frac{V_C}{rci0} \frac{1}{\sqrt{1 + \left( \frac{V_C}{V_{lim}} \right)^2}} \left\{ \begin{array}{ll} 1 & V_C \leq V_{lim} \\ \frac{V_C - V_{lim}}{V_{PT}} & V_C > V_{lim} \end{array} \right. \quad V_C = V_{CiEi} - V_{CES}$$

The plot on the right illustrates the limiting cases for small and large $V_{CE}$ values. The equation above has discontinuous derivatives at $V_C = V_{lim}$ and returns negative $I_{CK}$ in $V_C < 0$. Thus it is not compact model compliant in this form.

Fig. 3.21 of [2]
The critical current, low voltage asymptote

The HICUM formulation connects the low and high voltage sections as seen below.

I. Linking to small \( V_{ciei} \)

\[
v_{ces_{eff}} = v_{ces} + V_T
\]

\[
u_{vc} = \frac{V_{ciei} - v_{ces_{eff}}}{V_T}
\]

\[
v_{ceff} = V_T \left[ 1 + \frac{1}{2} \left( u_{vc} + \sqrt{u_{vc}^2 + 1.921812} \right) \right]
\]

This construct converges to \( V_T \) when \( V_{ciei} \) tends to \(-Inf\). The smoothing function modifies the saturation voltage by an additional \( V_T \) term. It implies an unphysical temperature dependence as well

\[
v_{ces_{eff}}(T) = vces(T) + \frac{kT}{q} = vces(T_0)(1 + \alpha_{Ces} \Delta T) + V_{T0} \left( 1 + \frac{\Delta T}{T_0} \right)
\]

\[
\alpha_{Ces,eff} = \frac{vces(T_0)\alpha_{Ces} + \frac{k}{q}}{vces(T_0) + V_{T0}}
\]

The physical \( \alpha_{Ces} \) has to be tweaked for achieving a fit to measurements.
The critical current, high voltage asymptote

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II. Linking to high $V_{ciei}$

Link variable: $x = \frac{v_{ceff} - V_{lim}}{V_{PT}}$  
Full critical current: $I_{CK} = \frac{v_{ceff}}{rc_i 0} \left( 1 + \frac{x + \sqrt{x^2 + a_{ck}}}{2} \right) \left( 1 + \left( \frac{v_{ceff}}{V_{lim}} \right)^{\delta_{ck}} \right)^{-1/\delta_{ck}}$

For sufficiently large $V_{CIEI}$: $x \approx \frac{V_{ciei} - V_{lim}}{V_{PT}}$  
$x$ is small around $V_{ciei} \approx V_{lim} \implies$ yielding: $V_{ciei} \approx V_{lim}$

$a_{ck} << x^2 : I_{CK} \approx \frac{V_{lim}}{2^{1/\delta_{ck}} rci 0} \left( 1 + \frac{x + |x|}{2} \right)$;  
$a_{ck} >> x^2 : I_{CK} \approx \frac{V_{lim}}{2^{1/\delta_{ck}} rci 0} \left( 1 + \frac{x + \sqrt{a_{ck}}}{2} \right)$

The derivative of the left eqn. is discontinuous at $V_{ce} = V_{lim}$

Accordingly, a kink was experimentally found in [3] using the model constant $a_{ck} = 0.001$

It has been made a model parameter since then
The critical current, $V_{ciei}$ masking

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In the range $0 \leq V_{ciei} \leq vces_{eff}$, $V_{CIEI}$ is masked by $vces_{eff}$ implying a pad in $vce_{eff}$.

This constant section propagates to the $I_{CK}$ function as a smoothing related part hence the physical description can be regarded valid only in the interval $V_{ciei} > vces_{eff}$.

$$V_{ciei} = vces_{eff} \rightarrow u_{vc} = 0$$

$$v_{eff} (0) = V_T (1 + 0.5 \sqrt{a_{vceff}})$$

$v_{eff,nom} (0) = 43.7 \text{mV}$

with reasonable neglections:

$$I_{CK0} \approx \frac{v_{eff} (0)}{rci0} \left( 1 + \frac{a_{ick}}{2} \right)$$

$rci0 = 20.61$ yields $I_{CK0} \approx 2.2 \text{mA}$.

$I_{CK0}$ is the smallest physical point on the $I_{CK}$ curves. The portion below is a smoothing created mathematical extension.
Implication on FO and FT

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The plots on the left show the effect of the multiples 1, 2 of $v_{ces}=65.11\text{mV}$ on the lines $V_{BE}=0.86\text{V}, 0.87\text{V}, 0.88\text{V}$ of the $V_{be}$ controlled output characteristics. The corresponding effective values are $v_{ces\,eff}=[91, 156]\text{mV}$. The blue markers intersect the largest curvature points where the true physical description starts. The closer the intercepts are to the $x$-axis, the more realistic the model.

FT is also affected: decreasing $V_{ce}$ decreases $I_{C}$ while $I_{CK}$ remains appr. constant.
Bump detail on the simulated $fo$ curve

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„In the region of peak of $fT$, the output characteristic presents always a bump. [4]” The simulated $go$ curve exhibits a local minimum at $V_{ce}=0.12$V marking a „kink” on the parent $fo$ curve. The latter takes an s-shaped form what is absent in the measurement. The $-0.2V<V_{ce}<0.12V$ simulated section is mathematical rather than physical.
Logexp smoothing

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Based on the generic form $V_c = V_{ciei} - vces$ a piecewise function can be prescribed

$$V_{ceff} = \begin{cases} 
  f_{minv} \cdot V_T & V_{ciei} \leq vces \\
  V_c & V_{ciei} > vces 
\end{cases}$$

merged by logexp smoothing

$$V_{ceff} = f_{minv} \cdot V_T + (f_{knee} \cdot V_T) \cdot \ln \left( 1 + \exp \left( \frac{V_c - f_{minv} \cdot V_T}{f_{knee} \cdot V_T} \right) \right)$$

Verilog-A realization:

**HICUM**

```
begin : HICICK
real d1, vceff, Vc2Vlim, ICK_ohm, FF_ick, ICK_low, vick_VPT;
d1 = vc*OVT-1.0;
vceff = (1.0+((d1+sqrt(d1*d1+`DFa_fj))/2.0))/VT;
Vc2Vlim = vceff/vlim_t;
c_ohm = c_ohm*Orc0_t;
FF_ick = exp(ln1+exp(delck*ln(Vc2Vlim)))/delck);
c_ohm = c_ohm/FF_ick;
vick_VPT= (vceff-vlim_t)/vpt;
ick = ICK_low*(1.0+0.5*(vick_VPT+sqrt(vick_VPT*vick_VPT+aick)));
end
```

**Proposed**

```
begin : HICICK
real d1, vceff, Vc2Vlim, ICK_ohm, FF_ick, ICK_low, vick_VPT;
real f_minv, f_knee, d2, arg;
f_minv = 0.001;
f_knee = 1.3;
d1 = f_minv*VT;
d2 = f_knee*VT;
arg = (vc-d1)/d2;

Vc2Vlim = vceff/vlim_t;
c_ohm = vceff*Orc0_t;
FF_ick = exp(ln1+exp(delck*ln(Vc2Vlim)))/delck);
c_ohm = c_ohm/FF_ick;
vick_VPT= (vceff-vlim_t)/vpt;
nick = ICK_low*(1.0+0.5*(vick_VPT+sqrt(vick_VPT*vick_VPT+aick)));
end
```

logexp smoothing is adapted in HICUM for the VPT affected capacitances with $Cexp\_lim=80$. The assigned $f_{minv}$, $f_{knee}$ values were found experimentally. $f_{knee}$ adjusts the sharpness of the junction of the two branches.
Comparision

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The conventional (HICUM) and the proposed logexp smoothed curves are compared with the same parameters as those on slide#7. The logexp lines tend to an asymptote of $0.001VT$ on the negative axis. If the $f_{knee}$ parameter is smaller the knee radius reduces which may imply a convergence sensitive situation. The indicated value as providing a larger radius than the approved reference appears to be a safe compromise.
Examples

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$$lb = 2.1332e-007; 4.2664e-007; 1.0666e-006; 2.1332e-006; 4.2664e-006; 1.0666e-005; 2.1332e-005$$

Bumps appear as sharp $go$ minimums in $Vce >= 0$ (see slide #9)

Top: hyperbolic smoothing, card1 [5]
Bottom: logexp smoothing, card1 [5]

Bumps disappeared from the useful $Vce > 0$ domain
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$$lb=2.1332e-007;4.2664e-007;1.0666e-006;2.1332e-006;4.2664e-006;1.0666e-005;2.1332e-005$$

Bumps appear as sharp $go$ minimums in $Vce\geq0$ (see slide #9)

Top: hyperbolic smoothing, card2 [5]  
Bottom: logexp smoothing, card2 [5]  

Bumps disappeared from the useful $Vce>0$ domain
Summary

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- the analysis revealed that the $vces$ parameter is increased in the model by $V_T$
- consequently the TC of this parameter also deviates from its physical value
- a theoretical explanation was given for the $V_{lim}$ kink discovered in [3]
- a formula was derived for the smallest physical $I_{CK}$ points termed $I_{CK0}$
- implications of $I_{CK0}$ on the forward output and FT characteristics were demonstrated
- logexp smoothing was proposed to push $I_{CK0}$ to a very small value
- a Verilog-A code snippet was given for the implementation and tests
- the validation was made using verified HICUM/L2 cards
- the bumps are suppressed and the physical $vces$ and its TC are restored
Acknowledgement
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Thank is due to D. Celi, STM for the field proven HICUM/L2 test cards!
References

Removing the $I_{CK}$ implied bumps in HICUM


Thank you!

Please visit our website
www.ams.com