A new transit time extraction algorithm based on matrix deembedding techniques

C. Raya, N. Kauffmann, D. Celi, T. Zimmer
# Introduction

**T<sub>F</sub> importance:**
- T<sub>F</sub> physical information from S parameters
- T<sub>F</sub> necessary to extract all high injection model parameters
- T<sub>F</sub> more convenient than using F<sub>T</sub> (Effects of transit time and capacitances decoupled)
- Less computational effort for extraction / optimization

**Brief review of a few existing methods:**
- Conventional method based on 1/(2.π.F<sub>T</sub>)
- Methods based on de-embedding techniques

**Objective:**
- Accurate transit time extraction including the saturation at high injection
- Focus on the deep saturation region especially for advanced BiCMOS technologies
- Figures of merit to help improve extraction strategy (e.g. T<sub>BVL</sub> & D<sub>ToH</sub> extraction, capacitance behavior in the forward region)
Outline

I – State of art

II – Improved Transit time extraction method

III – Extraction Results

IV - Conclusion
I – State of art
State of art  | Method  | Result
--- | --- | ---

**Conventional method: tangent method**

\[
\frac{1}{2\pi f_{\text{req}}.F_t} - \tau_{\text{cor}} = T_F + (C_{BC}(1 + 1/\beta_0) + C_{BE}). \frac{1}{g_m} \quad \text{with} \quad g_m \approx \frac{I_c}{m_c.V_T}
\]

\[
\tau_{\text{cor}} = (r_E + r_{CX}).C_{BC}(1 + 1/\beta_0)
\]

![Graph showing the relationship between $T_F$, $F_T$, and $V_T/Ic$](image)

\[T_F = T_{F0} + \Delta T_F\]
### State of art Method Result

**Conventional method: tangent method**

<table>
<thead>
<tr>
<th>Vbc</th>
<th>Robust at low injection, although it gives effective extracted parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6V, 0.4V, 0.2V, 0V, -0.5V, -1V, -1.5V</td>
<td>T_F extraction not accurate especially at high injection and in the saturation region</td>
</tr>
<tr>
<td></td>
<td>Bad g_m approximation</td>
</tr>
</tbody>
</table>

![Graph](image-url)
State of art  Method  Result

M. Malorny, M. Schröter
“Analytical method for calculating elements of an arbitrary equivalent circuit”, MIXDES 2004, Poland, pp.79-84

Z. Huszka
“A Pragmatic Yet Accurate Transit Time Extraction Method for HICUM”, Private communication

2 Methods based on de-embedding techniques

- Good determination of $g_m$ and $g_0$
- Accurate $T_F$ extraction
- The saturation involves a small error for the high injection region.
- Requires a very good accuracy of the capacitances $C_{BE}$ and $C_{BC}$

The new suggested method is an improvement of this approach
II – Improved transit time extraction method
Strategy: Access to the $T_F$ dependent main circuit

Access to the $T_F$ dependent circuit by deembedding techniques requires to determine:

- The temperature
- The internal nodes
- Calculate all bias dependant components of the small signal circuit
II – Improved transit time extraction method

- Approximation of the internal temperature
- Approximation of the internal nodes B’, B*, E’, C’, E’, S’
- Component calculation of the equivalent small signal circuit

Full deembedding by matrix calculation

Transit time extraction
**Temperature**

Extraction from S-parameters measurement => Small signal => $C_{th}$ could be neglected

\[
\Delta T_j = T_{ambient} + P \cdot R_{th}
\]

\[
T_j = T_{ambient} + P \cdot R_{th}
\]

<table>
<thead>
<tr>
<th>State of art</th>
<th>Method</th>
<th>Result</th>
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<tbody>
<tr>
<td><strong>Temperature</strong></td>
<td>Extraction from S-parameters measurement =&gt; Small signal =&gt; $C_{th}$ could be neglected</td>
<td><strong>Result</strong></td>
</tr>
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<table>
<thead>
<tr>
<th>$F_{LSH} = 0$</th>
<th>$P = I_{c_{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{LSH} = 1$</td>
<td>$P \approx I_{c_{meas}} \cdot (V_{CE} \cdot R_{E} \cdot (I_{c_{meas}} + I_{b_{meas}}) - R_{CX} \cdot I_{c_{meas}})$</td>
</tr>
</tbody>
</table>
| $F_{LSH} = 2$ | $P_1 = I_{b_{meas}} \cdot (V_{BE} \cdot R_{E} \cdot (I_{c_{meas}} + I_{b_{meas}}))$  
$P_2 = I_{c_{meas}} \cdot (V_{CE} \cdot R_{E} \cdot (I_{c_{meas}} + I_{b_{meas}}) - R_{CX} \cdot I_{c_{meas}})$  
$P_3 = R_{E} \cdot (I_{c_{meas}} + I_{b_{meas}})^2$  
$P \approx P_1 + P_2 + P_3$ |

**Temperature approximation**

Verilog-A Code

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### Approximation of the internal nodes

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<tr>
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<tbody>
<tr>
<td>( r_{CX} )</td>
<td>( C' \rightarrow C )</td>
<td>( V_{C'} = V_C - r_{CX}(T_J) \cdot I_{C_{meas}} )</td>
</tr>
<tr>
<td>( r_{BX} )</td>
<td>( B \rightarrow B^* )</td>
<td>( V_{B^*} = V_B - r_{BX}(T_J) \cdot I_{B_{meas}} )</td>
</tr>
<tr>
<td>( r_{E} )</td>
<td>( E' \rightarrow E )</td>
<td>( I_e \approx I_{B_{meas}} + I_{C_{meas}} ) (Structure RF common emitter =&gt; ( I_e ) is not measured)</td>
</tr>
<tr>
<td>( r_{SU} )</td>
<td>( S' \rightarrow S )</td>
<td>For ( V_{BC} &lt; 0.5V ) the substrate current (( I_s ) is not measured) is low (( V_{SS'} \approx 0V )). ( V_{S'} ) is assumed equal to ( V_S ) (involving an error on the calculation of ( V_{S'C'} ) and ( V_{S'B^*} ))</td>
</tr>
<tr>
<td>( r_{bi^*} )</td>
<td>( B^* \rightarrow B' )</td>
<td>( I_{r_{bi^*}} = I_{B_{meas}} - I_{BC} - I_{BEp} - I_{ts} \cdot T_{UNODE} \cdot I_{BEtp} )</td>
</tr>
<tr>
<td>( r_{bi^*} ) calculated for a self-consistent algorithm</td>
<td></td>
<td></td>
</tr>
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</table>
II – Improved transit time extraction method

Approximation of the internal temperature
Approximation of the internal nodes B’, B*, E’, C’, E’, S’
Component calculation of the equivalent small signal circuit

Full deembedding by matrix calculation

Transit time extraction
Deembedding

**Y Matrix**

\[ [Y] = [Y_0] + [Y_1] \]
\[ [Y_0] = [Y] - [Y_1] \]

**A Matrix**

\[ [A] = [A_1] . [A_0] \]
\[ [A_0] = [A_1]^{-1} . [A] \]

**Z Matrix**

\[ [Z] = [Z_0] + [Z_1] \]
\[ [Z_0] = [Z] - [Z_1] \]
# Full deembedding

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<tr>
<td></td>
<td></td>
<td>A Step1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y Step2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A Step3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y Step4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z Step5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y Step6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A Step7 (*)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y Step8</td>
</tr>
</tbody>
</table>

(*) $r_{bi}$ is deembedded as a passive resistance ($r_{bi}$ is bias dependent)
Different approaches available to compute $r_{bi}$: calculation, extraction, ...

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II – Improved transit time extraction method

- Approximation of the internal temperature
- Approximation of the internal nodes B’, B*, E’, C’, E’, S’
- Component calculation of the equivalent small signal circuit

Full deembedding by matrix calculation

Transit time extraction
Y matrix (from HICUM Verilog-A code)

\[
\begin{bmatrix}
g_{avl_c} + g_{avl_b} + 2\pi.f_{\text{req}}\left( C_{dE} + C_{dE_{bc}} + C_{dC_{be}} + C_{dC} \right) & -g_{avl_c} - j.2\pi.f_{\text{req}}\left( C_{dE_{bc}} + C_{dC} \right) \\
g_{m} - g_{avl_c} - g_{avl_b} - j.2\pi.f_{\text{req}}\left( C_{dC} + C_{dC_{be}} \right) & g_{0} + g_{avl_c} + j.2\pi.f_{\text{req}}C_{dC}
\end{bmatrix}
\]

\[
\partial Q_f = C_{dE_{bc}} \frac{\partial Q_f}{\partial V_{B'E'}} + C_{dE_{bc}} \frac{\partial Q_f}{\partial V_{B'C'}}
\]

\[
\partial Q_r = C_{dC_{be}} \frac{\partial Q_r}{\partial V_{B'C'}} + C_{dC_{be}} \frac{\partial Q_r}{\partial V_{B'E'}}
\]

\[
C_{dE} = \left. \frac{\partial Q_f}{\partial V_{B'E'}} \right|_{V_{B'C'}=\text{cte}}
\]

\[
C_{dE_{bc}} = \left. \frac{\partial Q_f}{\partial V_{B'C'}} \right|_{V_{B'E'}=\text{cte}}
\]

\[
C_{dC} = \left. \frac{\partial Q_r}{\partial V_{B'C'}} \right|_{V_{B'E'}=\text{cte}}
\]

\[
C_{dC_{be}} = \left. \frac{\partial Q_r}{\partial V_{B'E'}} \right|_{V_{B'C'}=\text{cte}}
\]
### State of art

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<tbody>
<tr>
<td><strong>$g_m$ &amp; $g_0$ determination</strong></td>
<td></td>
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</table>

\[
\begin{bmatrix}
g_{avl_c} + g_{avl_b} + 2\pi.f_{req}.(C_{dE} + C_{dE_bc} + C_{dC_be} + C_{dC}) & -g_{avl_c} - j.2.\pi.f_{req}.(C_{dE_bc} + C_{dC}) \\
g_m - g_{avl_c} - g_{avl_b} - j.2.\pi.f_{req}.(C_{dC} + C_{dC_be}) & g_0 + g_{avl_c} + j.2.\pi.f_{req}.C_{dC}
\end{bmatrix}
\]

\[
\begin{align*}
g_m &= R(Y_{21}) + R(Y_{11}) \\
g_0 &= R(Y_{22}) + R(Y_{12})
\end{align*}
\]

$R(Y_{11})$ and $R(Y_{12})$ are too noisy and sensitive to the deembedding and the measurement accuracy. In general $|g_m| >> |g_{avl_b} + g_{avl_c}|$ and $|g_0| >> |g_{avl_c}|$ then $g_{avl_b}$ and $g_{avl_c}$ could be neglected:

\[
\begin{align*}
g_m &\approx R(Y_{21}) \\
g_0 &\approx R(Y_{22})
\end{align*}
\]

N.B.: Another solution is to determine a self-consistent expression for $g_0$ and $g_m$ using:

\[
\begin{align*}
g_{avl_b} &= -K_{avl}.(g_m + g_0) \\
g_{avl_c} &= -K_{avl}.g_0 + g_a
\end{align*}
\]
State of art  Method  Result

\( C_{\text{dC_be}} \) negligible

\[
\text{imag}(Y_{11}) \over j.2.\pi.f_{\text{req}} = C_{dE} + C_{dE_{-bc}} + C_{dC} + C_{dC_{-be}}
\]

Rigorously \( C_{dC_{-be}} = \text{imag}(Y_{21})/(j.2.\pi.f_{\text{req}}) - \text{imag}(Y_{22})/(j.2.\pi.f_{\text{req}}) \)

However \( \text{imag}(Y_{22}) \) is too sensitive to the substrate network, moreover \( C_{dC_{-be}} \) is low for \( V_{BC} < 0.7 \text{V} \), then practically \( C_{dC_{-be}} \) is negligible.

\[
\text{imag}(Y_{11}) \over j.2.\pi.f_{\text{req}} \approx C_{dE} + C_{dE_{-bc}} + C_{dC}
\]
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<tr>
<td><strong>$T_F$ from $\text{Imag}(Y_{11})$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Imag}(Y_{11}) \approx C_{dE} + C_{dE_bc} + C_{dC}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{dE} = \left[ \frac{\partial Q_F}{\partial v_{B_E'}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} + \left[ \frac{\partial I_{TF}}{\partial v_{B_E'}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} \cdot \left[ \frac{\partial Q_f}{\partial I_{TF}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} = \left[ \frac{\partial Q_{FF}}{\partial v_{B_E'}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} + (g_m + g_0).T_F$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{dE_bc} = \left[ \frac{\partial Q_F}{\partial v_{B_C'}} \right]<em>{v</em>{B_E'}}^{v_{B_E'}} + \left[ \frac{\partial I_{TF}}{\partial v_{B_E'}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} \cdot \left[ \frac{\partial Q_f}{\partial I_{TF}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} = \left( I_{TF} \cdot \frac{dT_{f0}}{dv_{B_C'}} + \left[ \frac{\partial Q_{FF}}{\partial v_{B_C'}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} \right) - g_0.T_F$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\left[ \frac{\partial Q_{FF}}{\partial v_{B_E'}} \right]<em>{v</em>{B_C'}}^{v_{B_C'}} = - \left[ \frac{\partial Q_{FF}}{\partial v_{B_C'}} \right]<em>{v</em>{B_E'}}^{v_{B_E'}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\partial Q_{FF}}{\partial v_{B_E'}} = \frac{\partial \text{Imag}(Y_{11})}{\partial v_{B_E'}} \approx g_m.T_F + I_{TF} \cdot \frac{dT_{f0}}{dv_{B_C'}} + C_{dC}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_f \approx \frac{1}{g_m} \left( \frac{\text{Imag}(Y_{11})}{j.2.\pi.f_{req}} - C_0 \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_0 = I_{TF} \cdot \frac{dT_{f0}}{dv_{B_C'}} + C_{dC}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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### Determination of $C_0$ from Imag($Y_{12}$) at low injection

$$-\frac{\text{imag}(Y_{12})}{j.2.\pi.f_{\text{req}}} = C_{dE\_bc} + C_{dC} = \left( I_{TF} \cdot \frac{dT_{f0}}{dv_{B\_C'}} + \frac{dQ_{FF}}{dv_{B\_C'}} \right)_{B'E'} - g_{0}T_{F} + C_{dC}$$

For the **low injection** region:

$$\frac{\partial Q_{FF}}{\partial v_{B\_C'}}_{B'E',ITF} \approx 0 \quad \text{and} \quad g_{0}.T_{F} \approx 0$$

Therefore:

$$-\frac{\text{imag}(Y_{12})}{j.2.\pi.f_{\text{req}}} = C_{dE\_bc} + C_{dC} \approx I_{TF} \cdot \frac{dT_{f0}}{dv_{B\_C'}} + C_{dC} = C_{0}$$

At **low injection** region (before the Ft peak):

$$C_{0} \approx -\frac{\text{imag}(Y_{12})}{j.2.\pi.f_{\text{req}}}$$
Determination of $C_0$ from $\text{Imag}(Y_{12})$ at high injection

At low injection and $V_{BC} < 0.4V$, $C_{dc} \approx 0$, then:

\[
\frac{\text{imag}(Y_{12})}{j.2.\pi.f_{\text{req}}} \approx i_{TF}, \frac{dT_0}{dv_{B'C'}} + C_{dc} \approx i_{TF}, \frac{dT_0}{dv_{B'C'}}
\]

\[
\frac{\text{imag}(Y_{12})}{j.2.\pi.f_{\text{req}}i_{TF}} \approx \frac{dT_0}{dv_{B'C'}} = D_{T0H}, \frac{dc}{dv_{B'C'}} + T_{BVL}, \frac{1}{C_{jci0}(T_j)} \cdot \frac{d(C_{jci}(T_j))}{dv_{B'C'}}
\]

$T_{BVL}$ and $D_{T0H}$ are extracted from a linear regression:

\[
f = \left( -\frac{\text{imag}(Y_{12})}{j.2.\pi.f_{\text{req}}i_{TF}} \cdot \frac{C_{jci0}(T_j)}{d(C_{jci}(T_j))} \right) = T_{BVL} - c^2.D_{T0H}
\]

\[
c = \frac{C_{jci0}(T_j)}{C_{jci}(T_j)}
\]

\[
T_{f0} = T_0 + D_{T0H}(c - 1) + T_{BVL}\left(\frac{1}{c} - 1\right)
\]

\[
\frac{dc}{dv_{B'C'}} = -\frac{c^2}{C_{jci0}(T_j)} \cdot \frac{d(C_{jci}(T_j))}{dv_{B'C'}}
\]
Determination of $C_0$ from $\text{Imag}(Y_{12})$ at high injection

For the high injection region (after the $F_t$ peak):

$$C_0 = I_{TF} \left[ \frac{d}{dV_{B'C'}} D_{T0H} + T_{BVL} \frac{1}{C_{jcl0(T_j)}} \cdot \frac{d(C_{jcl}(T_j))}{dV_{B'C'}} \right]_{\text{calc}} + C_{dc}^*$$

$$C_{dc}^* = -\frac{\text{Imag}(Y_{12})}{j.2.\pi.f_{\text{freq}}} \left. \right|_{\text{peak } F_t} - \left. I_{TF} \left[ \frac{d}{dV_{B'C'}} D_{T0H} + T_{BVL} \frac{1}{C_{jcl0(T_j)}} \cdot \frac{d(C_{jcl}(T_j))}{dV_{B'C'}} \right]_{\text{calc}} \right|_{\text{peak } F_t}$$

![Graphs showing the comparison between C0 extracted and C0 Verilog-A](image)
**Real improvement**

\[ T_f \approx \frac{\text{imag}(Y_{11} + Y_{12})}{j.2.\pi.f_{\text{freq}}g_m} \]

\[ T_f \approx \frac{\text{imag}(Y_{11}) - C_0}{j.2.\pi.f_{\text{freq}}g_m g_m} \]

Without \( C_0 \) the result is the same as the Malorny/Shrörter and Huszka methods.

Full matrix deembedding convenient but actually not necessary.

Real improvement: \( T_{BL} \) & \( D_{T0H} \) extraction and \( dT_{F0} \) calculation (\( C_0 \)).
II – Extraction Results
### Application on measurement

**Final transit time extraction**

<table>
<thead>
<tr>
<th>Extracted $T_F$ [ps]</th>
<th>$T_F$ [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Smoothed</td>
</tr>
<tr>
<td></td>
<td>Raw extraction</td>
</tr>
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</table>

ST-BICMOS9 technology (0.13 $\mu$m): $F_T = 160$ GHz, $F_{max} = 160$ GHz

<table>
<thead>
<tr>
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<th>Result</th>
</tr>
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<tr>
<td><strong>State of art</strong></td>
<td></td>
</tr>
<tr>
<td>Application on measurement</td>
<td>BICMOS9 $W_E=0.3 \mu m$, $L_E=3.70 \mu m$</td>
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**Base-emitter capacitance correction**

- $F_T$
- $F_T$ deembedded Matrix
- Tuning $V_{DE}/Z_E$
- $C_{BE}$ extracted
- $C_{BE}$ simulation
State of art Method Result

**Application on measurement**

BICMOS9 $W_E=0.3 \mu m$, $L_E=3.70 \mu m$

---

For $V_{BC} > 0.4 V \Rightarrow C_{dc} \neq 0$

$T_{BVL}$ and $D_{TOH}$ extraction
Application on measurement

- **Base-Collector capacitance correction**
  - $C_{BC}$ parameters **not** enough accurate for $V_{BC}>0$

**Method**

- **F_T** measurement
- **F_T** simulation
- Tuning $V_{DC}/Z_C$
- $C_{BC}$ extracted
- $C_{BC}$ simulation

**Result**

BICMOS9 $W_E=0.3\mu m$, $L_E=3.70\mu m$

---

After direct $T_{BVL}$ & $D_{T0H}$ extraction
Application on measurement

Final transit time extraction

BICMOS9 $W_E=0.3\mu m$, $L_E=3.70\mu m$
IV – Conclusion
Conclusion

- **Highly accurate** $T_F$ **extraction method including the saturation region**
  - Better approximation of $T_F$ (better than $C_{dE} = g_m \cdot T_F$)
  - Full matrix deembedding convenient but not necessary

- **$T_{BVL}$ & $D_{T0H}$ extraction from $Y_{12}$**
  - Extraction using a linear regression
  - Method mainly sensitive to the well known $C_{BC}$ capacitance only

- **Capacitance parameters optimization in forward mode**
  - FOM: deembeded $F_T$ after matrix deembedding
  - Capacitance optimization without modifying the capacitance in reverse mode

- **New approach for $r_{bi*}$ extraction (Not presented):**
  - $r_{bi*}$ extraction less sensitive to the avalanche phenomenon